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ESSAIS EN MACROÉCONOMIE SUR LES DYNAMIQUES À COURT TERME DE L'INFLATION ET LES MARCHÉS FINANCIERS

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YOROU TCHAKONDO

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RÉSUMÉ

Cette thèse comprend trois chapitres relatifs aux dynamiques à court terme de l'inflation et à l'impact des marchés financiers sur l'économie réelle.

Le premier chapitre propose un modèle d'équilibre général dynamique et stochastique (DSGE) qui incorpore une structure en boucle de production à côté du trend d'inflation positif, afin d'analyser les sources des dynamiques à court terme de l'inflation. Il s'agit principalement de développer pour la première fois dans la littérature, et en présence de ces deux ingrédients, une formulation générale de la courbe de Phillips néo-keynésienne où, l'inflation est exprimée comme une fonction des coûts marginaux réels et de l'inflation future anticipée. En se concentrant sur l'analyse de la pente de la courbe de Phillips, nous montrons que le trend d'inflation positif et la structure en boucle de production sont nécessaires pour expliquer la persistance de l'inflation observée dans les données. Cependant, sous des valeurs raisonnables du trend d'inflation, les inputs intermédiaires jouent un rôle beaucoup plus important que le trend d'inflation en ce qui concerne la persistance inflationniste.

Dans le deuxième chapitre, nous visons à approfondir notre compréhension des dynamiques à court terme de l'inflation. Pour ce faire, nous simulons un modèle DSGE qui intègre non seulement la structure en boucle de production et le trend d'inflation positif, mais aussi des frictions réelles comme la formation d'habitude de consommation, les coûts d'ajustement du capital et l'utilisation variable du capital. Les autocorrélations théoriques de l'inflation obtenues du modèle simulé sont ensuite confrontées à celles observées dans les données de l'économie américaine. Les conclusions de la démarche analytique du premier chapitre sont confirmées ici. En effet, nous trouvons d'une part que le trend d'inflation positif a un effet négligeable sur la persistance de l'inflation en présence des inputs intermédiaires. D'autre part, la structure en boucle de production fournit une meilleure explication de l'évidence empirique sur la persistance de l'inflation.

Le troisième chapitre étudie les interconnections entre les marchés financiers et l'économie réelle. Le cadre d'analyse est un modèle DSGE qui rend compte des interventions des ménages sur les marchés financiers, à travers le modèle d'évaluation des actifs financiers de Fama et French (2004). Par ailleurs, nous proposons une modélisation explicite des dynamiques des marchés financiers sur la base du mouvement brownien géométrique. Comme résultats, nous montrons que la consommation, l'output et l'investissement réagissent moins alors que l'inflation et le travail réagissent plus fortement au choc technologique ici, que dans le cas d'une économie où le secteur financier est ignoré. En outre, les effets négatifs d'un choc de politique monétaire restrictive sur l'output, la consommation, l'inflation, l'investissement et le travail sont beaucoup plus importants. Par ailleurs, nous trouvons qu'un choc positif aux marchés financiers exerce une pression à la baisse sur le taux d'intérêt nominal lorsque le coefficient beta du portefeuille d'actifs du ménage est positif. Enfin, le modèle DSGE avec le secteur financier reproduit mieux la plupart des caractéristiques de l'économe U.S., en particulier, les volatilités et autocorrélations des principales variables macroéconomiques ainsi que leurs corrélations avec l'output.

Mots-clés : Dynamiques de l'inflation, persistance de l'inflation, prix rigides, biens intermédiaires, trend d'inflation positif, production en boucle, CAPM, marchés boursiers, politique monétaire, choix de portefeuille, mouvement brownien géométrique.

ABSTRACT

This thesis consists of three chapters related to short-term dynamics of inflation and the impact of financial markets on the real economy.

The first chapter offers a dynamic stochastic general equilibrium (DSGE) model that incorporates a roundabout structure of production alongside a positive inflation trend, to analyze the sources of short-term dynamics of inflation. In presence of both ingredients, the main goal here is to develop for the first time in the literature, a general New Keynesian Phillips Curve (NKPC) formulation, where inflation is expressed as a function of real marginal costs and expected future inflation. Focusing in our analysis on the NKPC-slope coefficient, we show that both ingredients are necessary to account for inflation persistence observed in the data. However, under plausible values of trend inflation, intermediate goods play a more significant role shaping inflation persistence than trend inflation.

In the second chapter, we aim to deepen our understanding of short-term dynamics of inflation. To do so, we simulate a DSGE model that incorporates not only roundabout production and positive trend inflation, but also real frictions such as habit formation, capital adjustment costs and variable capital utilization. The theoretical autocorrelations of inflation obtained from the simulated model are then compared with those observed in the U.S. data. The findings of the analytical approach of the first chapter are confirmed here. In fact, we find first that the positive trend inflation appears to have a negligible impact on inflation persistence when allowing for roundabout production. Second, intermediate goods provide a better explanation of the empirical evidence on inflation persistence.

The third chapter explores the interconnections between financial markets and the real economy. The framework is a DSGE model that accounts for households interventions on financial markets, through the capital asset pricing model (CAPM) of Fama and French (2004). Moreover, we propose an explicit modelling of financial markets dynamics based on geometric brownian motion. As results, we show that consumption, output and investment react less to a technology shock, while the nominal interest rate, inflation and labor are responding more strongly, compared to the case where financial markets are ignored. Moreover, the negative effects of a tightening monetary policy shock on output, consumption, inflation, investment and labor are more significant. We also find that a positive financial markets shock exerts a downward pressure on the nominal interest rate when the beta coefficient of the assets portfolio is positive. Finally, we find that our model with a financial market sector is successful in reproducing most of the salient features of the U.S. economy, particularly, key macroeconomic volatilities, autocorrelations, and correlations with output.

Keywords: Inflation dynamics, inflation persistence, sticky prices, intermediate goods, positive trend inflation, roundabout production, CAPM, stock markets, monetary policy, portfolio choice, geometric brownian motion.

INTRODUCTION

Le principal défi auquel s'attaque notre thèse est d'aider non seulement à une meilleure mise en application de la politique monétaire, mais aussi, à un examen beaucoup plus approfondi des effets de cette dernière sur l'économie réelle. Cela passe par deux canaux de recherche. D'abord, un objectif de maîtrise des dynamiques à court terme de l'inflation par les banques centrales et les chercheurs, ce à quoi s'attèlent les deux premiers chapitres. Puis, une grande compréhension par la communauté scientifique de l'analyse des interconnections entre les marchés financiers et l'économie réelle que constitue l'objet du dernier chapitre.

Une relation structurelle clé dans la catégorie des modèles d'équilibre général dynamique et stochastique (DSGE) est la courbe de Phillips néo-keynésienne (NKPC). La NKPC a connu ces dernières années, plusieurs développements visant à mieux expliquer les dynamiques à court terme de l'inflation et à améliorer de manière générale notre compréhension de la politique monétaire. La NKPC standard est log-linéarisée autour d'un trend d'inflation nul. Cette hypothèse est d'une part, contrefactuelle à cause d'un taux d'inflation en moyenne positif enregistré par les économies industrialisées de l'après-guerre, et d'autre part, non anodine comme tente de démontrer un courant de littérature récent. Ascari (2004) par exemple, montre que le trend d'inflation positif pourrait affecter de manière significative les propriétés de court terme et de long terme des modèles à prix rigides, alors que Ascari et Ropele (2007) et Coibion et Gorodnichenko (2011) trouvent que même un trend d'inflation faible aurait un impact sur la politique monétaire optimale et les dynamiques des variables macroéconomiques.

Cette thèse explore entre autres, et ce, à travers les deux premiers chapitres, la NKPC et les sources des dynamiques de l'inflation dans une économie avec trend d'inflation positif. Mais pour la première fois dans ce type de littérature, nous traitons cette problématique en utilisant un modèle DSGE qui incorpore non seulement le trend d'inflation positif, mais aussi et surtout une structure en boucle de production. La prise en compte de la structure en boucle de production est motivée par le fait que les biens finaux sont devenus de plus en plus sophistiqués et complexes en termes de production, notamment de la période de l'entre-deux guerres à la période de l'après-guerre (voir, Basu et Taylor, 1999a, 1999b; Hanes, 1996, 1999; Huang, Liu et Phaneuf, 2004). Au début du dix-neuvième siècle, le panier de consommation du ménage était principalement composé de biens relativement non finis. Depuis lors, les économies industrialisées ont été caractérisées par une augmentation des relations inputs-outputs dans la production des types de biens entrant dans le panier de consommation finale. Huang, Liu et Phaneuf (2004) montrent que la part U.S. des inputs intermédiaires a augmenté de 0.3 - 0.4 durant la période de l'entre-deux guerres à 0.6 - 0.7 durant la période de l'après-guerre, tandis que diverses études estiment que cette part se situe entre 0.6 et 0.9 pour la période de l'après-guerre (voir, Basu, 1995; Bergin et Feenstra, 2000; Huang et Liu, 2001; Huang, Liu et Phaneuf, 2004; Nakamura et Steinsson, 2010).

Ainsi, le premier chapitre propose pour la première fois dans ce type de modèles DSGE, et en présence de la boucle de producton et du trend positif, une formulation générale de la NKPC où l'inflation est exprimée comme une fonction des coûts marginaux réels et de l'inflation future anticipée. Nous trouvons ici que les interactions entre les prix rigides, les inputs intermédiaires et le trend d'inflation positif ont une forte influence sur la sensibilité de l'inflation aux coûts marginaux réels. Toutefois, les biens intermédiaires semblent avoir un impact plus important sur le coefficient de la pente de la NKPC que le trend d'inflation positif suggéré par Ascari (2004).

Dans le même ordre d'idées, le deuxième chapitre est une extension du modèle DSGE à trend d'inflation positif de Ascari (2004). Nous bonifions ce modèle en intégrant non seulement la structure en boucle de production, mais aussi diverses frictions réelles comme la formation d'habitude de consommation, les coûts d'ajustement du capital et l'utilisation variable du capital. Ces frictions réelles standard dans la littérature DSGE, semblent incontournables dès lors qu'on veut analyser la persistance des variables macroéconomiques (CEE, 2005; Smets and Wouters, 2003). La démarche ici, consiste d'abord à générer les autocorrélations de l'inflation à partir du modèle simulé, et ensuite, à les comparer à celles observées dans les données de l'économie américaine. Il s'en suit que le trend d'inflation positif a un effet négligeable sur les dynamiques de l'inflation en présence des inputs intermédiaires, et que ces derniers donnent une meilleure explication de l'évidence empirique sur la persistance de l'inflation. Ces résultats confortent ainsi nos conclusions de la démarche analytique du premier chapitre.

Enfin, le troisième chapitre s'inscrit dans la deuxième problématique de notre thèse, à savoir, les interconnections entres les marchés financiers et l'économie réelle. Le point de départ de ce travail est la récente crise financière de 2008, qui a montré comment des chocs négatifs aux marchés financiers pouvaient se transformer en des conséquences néfastes pour l'économie réelle. Cette crise a aussi mis en évidence l'inefficacité des instruments traditionnels de la politique monétaire dans un contexte de taux d'intérêt proches de la borne zéro. D'où, la nécessité d'un plus grand intérêt pour les marchés financiers ainsi qu'une analyse plus poussée de leurs impacts sur l'économie réelle. Nous nous attelons à cela en proposant un cadre d'analyse DSGE qui intègre le modèle d'évaluation des actifs financiers, pour rendre compte des interventions des ménages sur les marchés financiers (voir, Markowitz, 1959; Sharpe, 1964; Lintner, 1965; Fama, 1996; et Fama et French, 2004). Le modèle d'évaluation des actifs financiers suppose que les individus détiennent un portefeuille d'actifs composé d'actifs sans risque, et d'actifs risqués disponibles sur les marchés boursiers. De plus, une modélisation explicite des dynamiques des marchés financiers est proposée en se basant sur le mouvement brownien géométrique (voir, Kendall et Hill, 1953; Osborne, 1959; Roberts, 1959; Samuelson, 1965; Black et Scholes, 1973; Barmish et Primbs, 2011; et Lochowski et Thagunna, 2013). Nos résultats suggèrent que la consommation, l'output et l'investissement réagissent moins, alors que l'inflation et le travail réagissent plus fortement au choc technologique ici, que dans le cas d'une économie où le secteur financier est ignoré. Aussi, les effets négatifs d'un choc de politique monétaire restrictive sur l'output, la consommation, l'inflation, l'investissement et le travail sont beaucoup plus importants. Par ailleurs, nous trouvons qu'un choc positif aux marchés financiers exerce une pression à la baisse sur le taux d'intérêt nominal lorsque le coefficient beta du portefeuille d'actifs du ménage est positif. Enfin, le modèle DSGE avec le secteur financier reproduit mieux la plupart des caractéristiques de l'économe U.S., en particulier, les volatilités et autocorrélations des principales variables macroéconomiques ainsi que leurs corrélations avec l'output. Par conséquent, les marchés financiers, et plus précisément les marchés boursiers mériteraient une attention particulière de la part des autorités monétaires et des chercheurs, lorsqu'on vise à mieux comprendre les tenants et aboutissants de la politique monétaire.

CHAPTER I

THE NEW KEYNESIAN PHILLIPS CURVE: INTERMEDIATE GOODS MEET POSITIVE TREND INFLATION

Abstract

What happens when intermediate goods meet positive trend inflation in a New Keynesian Phillips Curve (NKPC) model? Focusing on the slope coefficient on marginal cost, our analysis shows the effects are dramatic. Unlike the basic Calvo price-setting model which requires an extremely low frequency of price adjustment or backward-looking components to account for inflation persistence, our model with sticky prices, roundabout production and trend inflation does successfully so with a plausible frequency of price changes, and realistic values of trend inflation and share of intermediate inputs. While trend inflation plays a non negligible role in explaining inflation dynamics, accounting for roundabout production seems to be more important.

JEL classification: E31, E32.

Keywords: Inflation dynamics; sticky prices; intermediate goods; trend inflation.

1.1 Introduction

A key structural relationship in a large class of dynamic stochastic general equilibrium (DSGE) models is the so-called New Keynesian Phillips curve (NKPC). The NKPC has undergone several developments in recent years aimed at better tracking short-run inflation dynamics and improving our understanding of monetary policy more generally. The standard NKPC is log-linearized around a zero steady-state rate of inflation. This assumption is not only counterfactual since postwar industrialized economies have experienced positive inflation on average, it is not innocuous as a recent body of research tends to demonstrate. Ascari (2004), for instance, shows that positive trend inflation may significantly alter the short-run and long-run properties of sticky-price models, while Ascari and Ropele (2007) and Coibion and Gorodnichenko (2011) argue that even low trend inflation may affect optimal monetary policy and the dynamics of macro variables.

The present paper further explores the NKPC and the sources of inflation dynamics in an economy with positive trend inflation. But for the first time in this type of literature, we address this question using a DSGE framework in which intermediate goods meet positive trend inflation. Focusing in our analysis on the dynamic response of inflation to real marginal costs (Galí and Gertler, 1999; Ascari, 2004), to which we refer throughout the paper as *slope coefficient on marginal cost* or NKPC-slope coefficient, we show that both ingredients are necessary to account for the observed inertial behavior of inflation, but that under plausible values of trend inflation, intermediate goods play a more significant role shaping inflation dynamics than trend inflation.

A well known property of the basic new keynesian sticky-price model with zero trend inflation is that the NKPC is purely forward-looking in that current inflation depends on current real marginal costs and expected future inflation. As we discuss in Section 2 of the paper, the basic NKPC is hardly reconcilable with the inertial behavior of inflation unless assuming an implausibly long average waiting time between price adjustments, backward-looking components (Galí and Gertler, 1999; Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2007; Justiniano and Primiceri, 2008), or slow-moving (random walk) trend inflation (Cogley and Sbordone, 2008).

Here, we combine a roundabout production structure with non-zero steady-state inflation in Calvo's (1983) price-setting framework. Previous research has established that in order to generate a high, positive serial correlation of inflation as observed in the U.S. data, the basic NKPC requires assuming a very high probability of price non-reoptimization. Working from the Calvo sticky-price model of King and Watson (1996), Nelson (1998, Table 3) shows that this probability must be close to 0.9 to match inflation persistence. This in turn implies an average waiting time between price adjustments of 2.5 years or more, which clearly is counterfactual. With a subjective discount factor of 0.99, the NKPC-slope coefficient would have to be 0.012 more or less. If the probability of price non-reoptimization is set instead at the more conventional value of 0.75, the NKPC-slope coefficient increases to 0.086, which is 7 times larger than required to match inflation inertia.

The inability of the basic NKPC model to account for inflation persistence without assuming an implausibly low frequency of price adjustments has led researchers to incorporate mechanisms like rule-of-thumb behavior of price-setters, backward indexation of prices, and slow-moving trend inflation in new keynesian pricing models (Galí and Gertler, 1999; Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2007; Justiniano and Primiceri, 2008; Cogley and Sbordone, 2008).

Contrasting sharply with previous studies, our approach does not need to rely on such ingredients to be consistent with inflation persistence. Still, it is fully consistent with the optimizing behavior of households and firms. Our framework is one that exploits strong interactions between a roundabout production structure, sticky prices and positive trend inflation. Our use of a roundabout production structure is motivated by the fact that final goods have become more processed and increasingly sophisticated over time, especially from the interwar period to the postwar period (e.g., Basu and Taylor, 1999a, 1999b; Hanes, 1996, 1999; Huang, Liu and Phaneuf, 2004). In the early Twentieth Century, a household's consumption basket was primarily composed of relatively unfinished goods. Since then, industrialized economies have been characterized by increased roundaboutness in the production of typical goods entering the final consumption basket. [1] Huang, Liu and Phaneuf (2004) document that the U.S. share of intermediate inputs has risen from $0.3 - 0.4$ during the interwar period to $0.6 - 0.7$ during the postwar period, whereas a variety of studies evaluate that this share lies between 0.6 and 0.9 for the postwar period. [2]

Working from a state-dependent model with nominal price rigidity, Basu (1995) shows that combining input-output linkages between firms with small (menu) cost of changing prices can give rise to a multiplier for price stickiness (MPS): because all firms in the economy face sticky prices and use intermediate inputs, firms' pricing decisions become interconnected, so that the amount of price stickiness at the aggregate level may well exceed that observed at the individual firm level. Using a fully articulated DSGE model with nominal and real frictions, El Omari and Phaneuf (2011) provide quantitative evidence that the MPS may be an important source of inflation inertia and persistence in aggregate quantities in a Calvo wage-and-price-setting framework with zero steady-state inflation.

Ascari (2004) extends the standard new keynesian pricing model to account for positive trend inflation, showing that positive steady-state inflation can significantly flatten the NKPC while reducing the sensitivity of current inflation to the current output gap. For example, assuming an annualized trend inflation of

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1. Hanes (1996) reports that the share of crude material inputs in final U.S. output has decreased from 26% to only 6% from the early twentieth century to the end of the 1960's. Furthermore, based on the household budget surveys, he reports that the share of consumption expenditures on food (excluding restaurant meals) has declined from 44% at the turn of the Twentieth Century to 11.3% in 1986, while the share of the budget category "Other" including many complex goods such as automobiles has risen steadily from 17% to 45.8% during the same period.

2. Basu (1995) argues that this share can be as high as 0.8, Bergin and Feenstra (2000) assume that it lies between 0.8 and 0.9, whereas Huang and Liu (2001), Huang, Liu and Phaneuf (2004) and Nakamura and Steinsson (2010) assume a share of intermediate inputs of 0.7.

only 2% and an elasticity of substitution between differentiated goods of 10, he shows that the slope coefficient on marginal cost decreases by 30% relatively to the basic model with zero-trend inflation. With a 5% trend inflation, the NKPC-slope coefficient drops by 64%. However, trend inflation must reach 8% to bring the NKPC-slope coefficient down to 0.012. Such a high steady-state rate of inflation seems implausible since the U.S. economy has experienced an average rate of inflation of 3.57% over the years 1960-2011, and 5% during the 1960s and 1970s, a time of high inflation.

We develop a general NKPC formulation that encompasses four different models: *i*) the basic price-setting model with zero-trend inflation, *ii*) a model with sticky prices, positive trend inflation and no input-output linkages, *iii*) a model with sticky prices, zero-trend inflation and intermediate inputs, and finally, *iv*) a model with sticky prices, positive trend inflation and intermediate inputs. We show that in all four models, inflation is expressed as a function of real marginal costs and expected future inflation, with a slope coefficient which is analytically and quantitatively different among the four models.

We provide evidence showing that the interactions between intermediate inputs, sticky prices and positive trend inflation exert a powerful impact on the response of inflation to real marginal costs. For instance, for a *median* waiting time between price adjustments of 7.2 months, broadly consistent with micro-level evidence on the behavior of prices (Bils and Klenow, 2004; Nakamura and Steinsson, 2008), we find that an annualized steady-state rate of inflation of only 1%, combined with a share of intermediate inputs of 0.6, reduce the slope coefficient on marginal cost to 0.029, which is nearly 66% lower than in the basic model with zero-trend inflation and no intermediate inputs. This reduction reaches 81% when trend inflation is 4%. But more importantly, with a rate of trend inflation between 3 to 5% and a share of intermediate inputs between 0.6 and 0.8, the NKPC-slope coefficient is always small, ranging from 0.006 to 0.02, which is broadly consistent with observed inflation dynamics.

However, among these two factors, intermediate inputs seem to have a larger impact on the NKPC-slope coefficient than positive trend inflation. That is, assuming a share of intermediate inputs of 0.6 in a model with sticky prices and zero steady-state inflation lowers the slope coefficient by roughly 60% relative to the basic model. Hence, the MPS is by itself an important channel of inflation persistence. However, we also show that with zero trend inflation, the share of intermediate inputs must be 0.8 or higher to bring the slope coefficient down to 0.012. Thus, while playing a smaller role than the MPS in reducing the NKPC-slope coefficient, taking into account positive trend inflation is nonetheless important to explain inflation dynamics. Furthermore, even when the frequency of price adjustments is set at a higher pace, we find that intermediate inputs and positive trend inflation have a powerful impact on the NKPC.

The rest of the paper is organized as follows. Section 2 traces back the various incarnations the NKPC has taken over the years, while giving some perspective on other approaches which have been followed to study inflation dynamics. Section 3 describes our DSGE model with sticky prices, roundabout production and positive trend inflation. Section 4 derives our general NKPC formulation and compares NKPC-slope coefficients in alternative models. Section 5 discusses calibration issues and presents our main findings. Section 6 offers concluding remarks.

1.2 The Various Incarnations of the NKPC

This section succintly analyzes the several incarnations the NKPC has gone through during the last fifteen years or so.[3]

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3. We purposefully restrict our analysis to new keynesian models involving nominal price stickiness.

1.2.1 The Forward-Looking NKPC

The standard Calvo sticky-price formulation implies a NKPC of the form:

$$\pi_t = \lambda mc_t + \beta E_t \{\pi_{t+1}\}, \tag{1.1}$$

where π_t indicates the inflation rate, mc_t denotes the firm's real marginal cost, and the slope coefficient on marginal cost, $\lambda \equiv (1-\xi_p)(1-\beta\xi_p)/\xi_p$, depends on the probability of price non-reoptimization ξ_p and the subjective discount factor β. Because the NKPC (1.1) is forward-looking, the Calvo price-setting model must rely on a high probability of price non reoptimization ξ_p (resulting in a low λ) to account for inflation persistence. Nelson (1998) shows that the standard Calvo sticky-price model generates high serial correlations of inflation as observed in the U.S. data with $\xi_p = 0.9$, implying an average waiting time between price adjustments of 2.5 years. This is implausibly long in light of microeconomic evidence on U.S. price behavior suggesting a median waiting time between 4.3 and 9 months for price changes (Bils and Klenow, 2004; Nakamura and Steinsson, 2008).

A subsequent development by Woodford (2003, ch.3; 2005) incorporates firm-specific (immobile) capital and variable demand elasticity into the otherwise standard Calvo sticky-price model. The marginal cost of the optimizing firm then differs from aggregate marginal cost by a function of its relative price. Denoting the elasticity of substitution among differentiated goods by θ, and the elasticity of marginal cost to firms' output by χ, the NKPC becomes:

$$\pi_t = \lambda_c mc_t + \beta E_t \{\pi_{t+1}\}, \tag{1.2}$$

where $\lambda_c \equiv (1-\xi_p)(1-\beta\xi_p)/\xi_p(1+\theta\chi)$. This formulation accommodates a lower slope coefficient on marginal cost for a given value of ξ_p. That is, the higher $\theta\chi$, the lower λ_c, and the weaker is the response of inflation to real marginal cost. Therefore, the probability of price non-reoptimization does not have to be as high as in the basic model to account for inertial inflation. [4]

4. See Eichenbaum and Fisher (2007) for an empirical investigation of the Calvo pricing model with firm-specific capital.

1.2.2 Backward-Looking Elements and the NKPC

One important development, initiated by the work of Galí and Gertler (1999), is the addition of backward-looking components to the NKPC intended to capture the inertial behavior of inflation. Galí and Gertler propose a variant of the Calvo pricing model in which firms facing the signal $1-\xi_p$ authorizing price changes are divided in two groups. One group of firms, in proportion ω, sets prices equal to the average price in the most recent round of price adjustment, plus a correction for last period rate of inflation. The other group, in proportion $(1-\omega)$, sets prices optimally as in the basic, forward-looking price-setting model. This refinement leads to the following *hybrid* NKPC:

$$\pi_t = \lambda_h mc_t + \gamma_f E_t \{\pi_{t+1}\} + \gamma_b \pi_{t-1}, \tag{1.3}$$

where $\lambda_h \equiv (1-\omega)(1-\xi_p)(1-\beta\xi_p)\varphi^{-1}$, $\gamma_f \equiv \beta\xi_p\varphi^{-1}$, $\gamma_b \equiv \omega\varphi^{-1}$ and $\varphi \equiv \xi_p + \omega\left[1-\xi_p(1-\beta)\right]$. The presence of rule-of-thumbers has three main consequences: it adds previous period inflation to the NKPC, lowers the slope coefficient on marginal cost and reduces the impact of expected future inflation on current inflation. Galí and Gertler (1999) report estimates suggesting that the backward-looking term in (1.3) is statistically significant and relatively modest, helping the hybrid formulation to better capture inflation dynamics.

In the same vein, Christiano, Eichenbaum and Evans (2005) propose a setup where firms which are not allowed to reoptimize their price in a given period will nonetheless index them to last period inflation. The remaining firms reset prices optimally as in the standard model.[5] The resulting NKPC is similar to (1.3), except that the coefficient on previous period inflation depends upon the degree of backward indexation. CEE argue that backward indexation helps reproduce the impulse responses of inflation and output to a monetary policy shock estimated from a structural vector autoregression.

The use of backward-looking terms has been subject to criticism. Woodford

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5. More precisely, in CEE's model, both households and firms not authorized to reoptimize their wages and prices, respectively, in a given period will index them to last past period inflation.

(2007), Cogley and Sbordone (2008) and Chari, Kehoe and McGrattan (2009) note that rule-of-thumb behavior of price-setters and backward indexation both lack a convincing microeconomic justification and are therefore *ad hoc* mechanisms. Moreover, both mechanisms have unattractive empirical implications. While the estimates reported in Galí and Gertler (1999) suggest that rule-of-thumb behavior is modest, the frequency of price adjustments implied by the hybrid model remains low and far from micro level evidence. Backward indexation, on the other hand, implies that all firms change their prices once every three months, which is counterfactual.

1.2.3 Trend Inflation and the NKPC

While the above relationships are derived for a log-linearization around zero-trend inflation, a recent strand of literature imposes log-linearizing the non-linear equilibrium conditions of the Calvo model around a steady state with a time-varying trend inflation (Cogley and Sbordone, 2005, 2008; Ireland, 2007). This refinement leads to the following NKPC:

$$\widetilde{\pi}_t = \lambda_{tv}\widetilde{mc}_t + a_{1t}\mathrm{E}_t\left\{\widetilde{\pi}_{t+1}\right\} + a_{2t}\sum_{j=2}^{\infty}\psi_{1t}^{j-1}\mathrm{E}_t\left\{\widetilde{\pi}_{t+j}\right\}. \tag{1.4}$$

Here the symbol $\widetilde{}$ over a variable denotes a log-deviation from trend value, and hence $\widetilde{mc}_t = mc_t - \overline{mc}_t$ and $\widetilde{\pi}_t = \pi_t - \overline{\pi}_t$, where $\overline{mc}_t$ and $\overline{\pi}_t$ are trend variables, and λ_{tv}, a_{1t} and a_{2t} are time-varying parameters evolving with trend inflation. Furthermore, Cogley and Sbordone (2008) assume strategic complementarity, so these parameters also depend on θ and χ. Note that Cogley and Sbordone's original formulation is even more general than (1.4), since it also embeds backward indexation. However, we omit the backward-looking component for the reasons above, and because it is found to be statistically insignificant when time-varying trend inflation is also taken into account (see, Cogley and Sbordone, 2008). Cogley and Sbordone model trend inflation as a driftless random-walk. Their estimates imply a mean duration of prices which roughly consistent with the evidence reported in Bils and Klenow (2004). Despite the merits of this approach, West (2007) questions the use of the random walk as a way of modeling trend inflation, arguing that there is no economic rationale offered for this assumption. He therefore concludes that (1.4), like the NKPC with backward-looking components, relies on exogenous rather than intrinsic sources of inertia.

Ascari (2004) and Ascari and Ropele (2007) consider the case of a constant, nonzero steady-state rate of inflation in a purely forward-looking price-setting model. Trend inflation is directly linked to monetary policy through the gross steady-state growth rate of money supply denoted by γ. The NKPC with positive trend inflation is:

$$\pi_t = \lambda(\gamma) mc_t + \beta \mathrm{E}_t \{\pi_{t+1}\} + (1-\gamma) F(\mathrm{E}_t \pi_{t+i}, \mathrm{E}_t y_{t+i}), \tag{1.5}$$

where $\lambda(\gamma) = \left(\frac{1-\xi_p \gamma^{\theta-1}}{\xi_p \gamma^{\theta-1}}\right)(1-\xi_p \beta \gamma^{\theta})$ and F is a function of expected future inflation and output. With zero steady-state inflation, $\gamma = 1$, and (1.5) bolds down to the basic NKPC. The slope coefficient on marginal cost, $\lambda(\gamma)$, is now a function of positive trend inflation. Assuming $\beta = 0.99$, $\theta = 10$ and $\xi_p = 0.75$, Ascari (2004) shows that $\lambda(\gamma)$ is smaller than λ by 30% with an annualized trend inflation rate of 2%, by 64% with 5% trend inflation and by 95% if trend inflation is 10%. Thus, positive trend inflation may significantly affect inflation dynamics.

1.3 A NKPC Model with Intermediate Goods and Positive Trend Inflation

Since our focus is on the NKPC and the slope coefficient on marginal cost, we follow Galí and Gertler (1999) and limit our modeling strategy to an environment of monopolistically competitive firms facing sticky prices. Our model rests on three main pillars. First, the production structure reflects the reality that many goods produced in industrialized economies have become increasingly processed over time (e.g., see Basu, 1995; Huang, Liu and Phaneuf, 2004). We model the increased sophistication of goods produced as a roundabout process, where all firms use intermediate inputs in production. Basu (1995) endorses the roundabout production structure based on the evidence from input-output studies showing that "even the most detailed input-output tables show surprisingly few zeros" (p.514). Second, prices are sticky. As Basu suggests, when combined with sticky prices, intermediate goods can act as a multiplier for price stickiness (MPS): a given amount of price rigidity at the individual firm level may lead to a higher degree of price stickiness at the aggregate level. Third, following Ascari (2004) and Ascari and Ropele (2007), we assume positive trend inflation.

1.3.1 Optimal Pricing Decisions

Denote by X_t a composite of differentiated goods $X_t(j)$ for $j \in [0,1]$ such that $X_t = [\int_0^1 X_t(j)^{(\theta-1)/\theta} dj]^{\theta/(\theta-1)}$, where $\theta \in (1,\infty)$ is the elasticity of substitution between the goods. The composite good is produced in a perfectly competitive aggregate sector.

The demand function for good of type j resulting from optimizing behavior in the aggregation sector is given by

$$X_t^d(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\theta} X_t, \tag{1.6}$$

where P_t is the price of the composite good related to the prices $P_t(j)$ for $j \in [0,1]$ of the differentiated goods by $P_t = [\int_0^1 P_t(j)^{(1-\theta)} dj]^{1/(1-\theta)}$.

The central feature of the model is that the composite good can serve either as a final consumption or investment good, or as an intermediate production input. The production of good j requires the use of capital, labor, and intermediate inputs:

$$X_t(j) = \Gamma_t(j)^{\phi}[K_t(j)^{\alpha} L_t(j)^{1-\alpha}]^{1-\phi} - F, \tag{1.7}$$

where $\Gamma_t(j)$ is the input of intermediate goods, $K_t(j)$ is the physical capital stock, $L_t(j)$ denotes total hours worked, and F is a fixed cost which is identical across firms. The parameter $\phi \in (0,1)$ measures the elasticity of output with respect to intermediate input, and the parameters $\alpha \in (0,1)$ and $(1-\alpha)$ are the elasticities of value-added with respect to the capital and labor input, respectively.

Each firm acts as a price-taker in the input markets and as a monopolistic competitor in the product market. A firm chooses the price of its product, taking the demand schedule in (1.6) as given. Prices are set according to the mechanism spelled out in Calvo (1983). In each period, a firm faces a constant probability $1-\xi_p$ of reoptimizing its price, with the ability to reoptimize being independent across firms and time.

A firm j allowed to reset its price at date t chooses a price $P_t(j)$ that maximizes its profits,

$$E_t \sum_{\tau=t}^{\infty} (\xi_p)^{\tau-t} D_{t,\tau}[P_t(j)X_\tau^d(j) - V(X_\tau^d(j))], \tag{1.8}$$

where E is an expectations operator, $D_{t,\tau}$ is the price of a dollar at time τ in units of dollars at time t and $V(X_\tau^d(j))$ is the cost of producing $X_\tau^d(j)$, equal to $V_\tau[X_\tau^d(j)+F]$, and V_τ denotes the marginal cost of production at time τ.

Solving the profit-maximization problem yields the following optimal pricing decision rule:

$$P_t(j) = \left(\frac{\theta}{\theta-1}\right)\left[\frac{E_t \sum\limits_{\tau=t}^{\infty} (\xi_p)^{\tau-t} D_{t,\tau} X_\tau^d(j) V_\tau}{E_t \sum\limits_{\tau=t}^{\infty} (\xi_p)^{\tau-t} D_{t,\tau} X_\tau^d(j)}\right], \tag{1.9}$$

which says the optimal price is a constant markup over a weighted average of marginal costs during the periods the price will remain effective.

Solving the firm's cost minimization problem yields the following nominal marginal cost function:

$$V_\tau = \overline{\phi} P_\tau^{\phi}[(R_\tau^k)^{\alpha} W_\tau)^{1-\alpha}]^{1-\phi}, \tag{1.10}$$

where R_τ^k is the nominal rental rate on capital, W_τ is the aggregate nominal wage rate and $\overline{\phi}$ is a constant term determined by ϕ and α. The nominal marginal cost records three components. Two of those are flexible, R_τ^k and W_τ, while the other, P_τ, is rigid since prices are sticky. The relative importance of the rigid price P_τ increases with the share of intermediate inputs ϕ.

Real marginal cost is therefore expressed as:

$$MC_{r\tau} = \left(\frac{V_\tau}{P_\tau}\right) = \overline{\phi}\left[\left(r^k_\tau\right)^\alpha (w_\tau)^{1-\alpha}\right]^{1-\phi}, \tag{1.11}$$

with $r^k_\tau = R^k_\tau / P_\tau$ and $w_\tau = W_\tau / P_\tau$. The higher the share of intermediate inputs ϕ, the smaller the impact of the two flexible components r^k_τ and w_τ on real marginal cost. Thus, real marginal becomes increasingly sluggish as ϕ rises, enhancing inflation persistence. With $\phi \to 1$, real marginal cost becomes almost insensitive to variations in the real rental rate on capital and in the real wage.

In the standard Calvo price-setting model with no intermediate inputs ($\phi = 0$), the real marginal cost is:

$$MC_{s\tau} = \left(\frac{r^k_\tau}{\alpha}\right)^\alpha \left(\frac{w_\tau}{1-\alpha}\right)^{1-\alpha}. \tag{1.12}$$

The conditional demand functions for the intermediate input and for the primary factor inputs used in the production of $X^d_\tau(j)$ which are derived from cost-minimization are

$$\Gamma_\tau(j) = \phi \frac{V_\tau[X^d_\tau(j) + F]}{P_\tau}, \tag{1.13}$$

$$K_\tau(j) = \alpha(1-\phi)\frac{V_\tau[X^d_\tau(j) + F]}{R^k_\tau}, \tag{1.14}$$

and

$$L_\tau(j) = (1-\alpha)(1-\phi)\frac{V_\tau[X^d_\tau(j) + F]}{W_\tau}. \tag{1.15}$$

A firm that does not reset its price at a given date still has to choose the inputs of the intermediate good, capital and labor to minimize production cost.

The pricing equation (3.4) can be rewritten as:

$$P_t(j) = \left(\frac{\theta}{\theta-1}\right)\left[\frac{E_t \sum\limits_{\tau=t}^{\infty} (\xi_p)^{\tau-t} D_{t,\tau} MC_{r\tau} P^\theta_\tau X_\tau}{E_t \sum\limits_{\tau=t}^{\infty} (\xi_p)^{\tau-t} D_{t,\tau} P^{\theta-1}_\tau X_\tau}\right]. \tag{1.16}$$

From (1.11) and (1.12), we can establish the following log-linear relationship between the real marginal cost with roundabout production and its counterpart in the basic price-setting model:

$$\widehat{mc}_\tau = mc_{\tau t} = (1-\phi)mc_{s\tau}. \tag{1.17}$$

In the absence of intermediate inputs ($\phi = 0$), the real marginal cost is $\widehat{mc}_\tau = mc_{s\tau}$, and

$$mc_{s\tau} = \alpha r^k_\tau + (1-\alpha)w_\tau. \tag{1.18}$$

1.3.2 Monetary Policy

The government injects money into the economy through nominal transfers, so $T_t = M^s_t - M^s_{t-1}$ where M^s is the aggregate nominal money supply. Furthermore, following Ascari (2004) and Ascari and Ropele (2007), we assume that steady-state money supply evolves according to the fixed rule: $M^s_t = \gamma M^s_{t-1}$, where γ is the gross steady-state growth rate of nominal money supply and the source of positive trend inflation.

1.3.3 Equilibrium and Market-Clearing Conditions

An equilibrium consists of allocations $\Gamma_t(j)$, $K_t(j)$, $L_t(j)$ and price $P_t(j)$ for firm j, for all $j \in [0,1]$, together with prices $D_{t,t+1}$, P_t, R^k_t, and W_t, satisfying the following conditions: (i) taking the nominal wage rate and all prices but its own as given, each firm's allocations and price solve its maximization problem; (ii) markets for bonds, capital, labor and the composite good clear; (iii) monetary policy is as specified.

Along with (1.13), the market-clearing condition for the composite good

$$Y_t + \int_0^1 \Gamma_t(j)dj = X_t,$$

implies that equilibrium real GDP is related to gross output by

$$Y_t = X_t - \phi\frac{V_t}{P_t}\left[G_t X_t + F\right], \tag{1.19}$$

where $G_t \equiv \int_0^1 [P_t(j)/P_t]^{-\theta}dj$ captures the price-dispersion effect of staggered price contracts.

Meanwhile, the market-clearing conditions $\int_0^1 K_t^d(j)dj = K_t$ for capital and $\int_0^1 L_t^d(j)dj = L_t^d = L_t^s$ for labor, along with (1.14) and (1.15), imply that the equilibrium aggregate capital stock and labor are related to gross output by

$$K_{t-1} = \alpha(1-\phi)\frac{V_t}{R_t^k}[G_t X_t + F], \tag{1.20}$$

$$L_t = (1-\alpha)(1-\phi)\frac{V_t}{W_t}[G_t X_t + F]. \tag{1.21}$$

Equations (1.19), (1.20), and (1.21), together with the price-setting equation (1.9) characterize an equilibrium.

1.4 The NKPC: Intermediate Goods Meet Positive Trend Inflation

We now examine how intermediate goods and positive trend inflation interact to affect the NKPC. Our main focus is on the NKPC-slope coefficient or slope coefficient on marginal cost.

1.4.1 Optimal Pricing Decisions with Intermediate Goods and Non-Zero Trend Inflation

To see how intermediate goods and positive trend inflation affect the optimizing behavior of intermediate firms, we expand (1.16) and make explicit the contribution of cumulative gross inflation rates (CGIR) to price setting (e.g. see Ascari and Ropele, 2007) [6]:

$$P_t(j) = \left(\frac{\theta}{\theta-1}\right)\left[\frac{E_t\sum\limits_{\tau=t}^{\infty}(\xi_p)^{\tau-t}D_{t,\tau}P_t^{\theta}X_\tau(\Pi_{t+1}\times\Pi_{t+2}\times\ldots\times\Pi_\tau)^{\theta}MC_{r\tau}}{E_t\sum\limits_{\tau=t}^{\infty}(\xi_p)^{\tau-t}D_{t,\tau}P_t^{\theta-1}X_\tau(\Pi_{t+1}\times\Pi_{t+2}\times\ldots\times\Pi_\tau)^{\theta-1}}\right]. \tag{1.22}$$

6. The CGIR between time $t+1$ and τ is $\Pi_{t+1,\tau} = \Pi_{t+1}\times\Pi_{t+2}\times\ldots\times\Pi_\tau$, where $\Pi_\tau = P_\tau/P_{\tau-1}$.

Trend inflation enters (1.22) by setting $\Pi_\tau = \gamma$ for $\tau = t+1, t+2, ..., t+\infty$. In the basic model with zero trend inflation, $\gamma = 1$, and the CGIRs attached to future expected terms are equal to one at all times. Future expected terms are discounted by $\xi_p\beta$, β denoting a subjective discount factor. Working from a model without intermediate goods, Ascari (2004) and Ascari and Ropele (2007) show that positive trend inflation ($\gamma > 1$) has two important effects on the optimal pricing decisions of firms. A first effect is that CGIRs at different time horizons shift upwards, changing the effective discount factors $\xi_p\beta\gamma^\theta$ and $\xi_p\beta\gamma^{\theta-1}$ in the numerator and denominator, respectively. Thus, when intermediate firms are allowed to reoptimize their prices, they set them higher to prevent the erosion of relative prices and profits resulting from trend inflation. The second effect is that the future components in (1.22) are progressively multiplied by larger CGIRs, so that optimal pricing decisions with trend inflation reflect future economic conditions more than short-run cyclical variations.

After log-linearizing (1.22) around a steady state with a positive trend inflation and using (1.19) to account for value-added, we obtain after some algebraic manipulations:

$$\begin{aligned} p_{jt} - p_t \;=\; & E_t \sum_{\tau=t}^{\infty} (\xi_p\beta\gamma^\theta)^{\tau-t}(1-\xi_p\beta\gamma^\theta)(\theta\pi_{t,\tau} + y_\tau + \widehat{mc}_\tau) \\ & -E_t \sum_{\tau=t}^{\infty} (\xi_p\beta\gamma^{\theta-1})^{\tau-t}(1-\xi_p\beta\gamma^{\theta-1})[(\theta-1)\pi_{t,\tau} + y_\tau], \end{aligned} \tag{1.23}$$

where variables expressed in small characters denote log variables.

Combining (1.23) with the following log-linearized expression for the general price level:

$$p_{jt} - p_t = \left(\frac{\xi_p\gamma^{\theta-1}}{1-\xi_p\gamma^{\theta-1}}\right)\pi_t,$$

we obtain

$$\pi_t = \left(\frac{1-\xi_p\gamma^{\theta-1}}{\xi_p\gamma^{\theta-1}}\right)(1-\xi_p\beta\gamma^\theta)\widehat{mc}_t + \beta E_t\pi_{t+1} + (1-\gamma)F(E_t\pi_{t+i}, E_t y_{t+i}), \tag{1.24}$$

where

$$F(E_t\pi_{t+i}, E_t y_{t+i}) = \beta(1-\xi_p\gamma^{\theta-1})\{y_t - \left(\theta + \frac{\xi_p\gamma^{\theta-1}}{1-\xi_p\gamma^{\theta-1}}\right) E_t\pi_{t+1} \tag{1.25}$$
$$-(1-\xi_p\beta\gamma^{\theta-1})E_t\sum_{\tau=t}^{\infty}(\xi_p\beta\gamma^{\theta-1})^{\tau-t}[(\theta-1)\pi_{t+1,\tau+1} + y_{\tau+1}]\}.$$

1.4.2 The NKPC with Intermediate Goods and Non-Zero Trend Inflation

From (1.24), we obtain the following generalized NKPC formulation for an economy with roundabout production and positive trend inflation:

$$\pi_t = \lambda_{rti}\widehat{mc}_t + \beta\mathrm{E}_t\{\pi_{t+1}\} + (1-\gamma)F(\mathrm{E}_t\pi_{t+i}, \mathrm{E}_t y_{t+i}), \tag{1.26}$$

where the slope coefficient of the NKPC is given by

$$\lambda_{rti} = \left(\frac{1-\xi_p\gamma^{\theta-1}}{\xi_p\gamma^{\theta-1}}\right)(1-\xi_p\beta\gamma^{\theta}). \tag{1.27}$$

Replacing $\widehat{mc}_t$ in (1.26) by (1.17), we can express the NKPC as a function of real marginal cost in the basic pricing model, mc_{st}, hence easing comparisons between alternative models:

$$\pi_t = \lambda(\gamma,\phi)mc_{st} + \beta\mathrm{E}_t\{\pi_{t+1}\} + (1-\gamma)F(\mathrm{E}_t\pi_{t+i}, \mathrm{E}_t y_{t+i}), \tag{1.28}$$

where

$$\lambda(\gamma,\phi) = \left(\frac{1-\xi_p\gamma^{\theta-1}}{\xi_p\gamma^{\theta-1}}\right)(1-\xi_p\beta\gamma^{\theta})(1-\phi).$$

This general formulation nests several specific models studied in the literature. The basic Calvo model with sticky prices abstracts from intermediate goods and assumes zero steady-state inflation ($\phi = 0$ and $\gamma = 1$), resulting into the following basic NKPC:

$$\pi_t = \lambda mc_{st} + \beta E_t\{\pi_{t+1}\}, \tag{1.29}$$

where the slope coefficient is,

$$\lambda = \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}.$$

A model without intermediate goods ($\phi = 0$), but including positive trend inflation ($\gamma > 1$), yields the following NKPC introduced by Ascari (2004) and Ascari and Ropele (2007):

$$\pi_t = \lambda(\gamma) mc_{st} + \beta E_t\{\pi_{t+1}\} + (1-\gamma) F(E_t \pi_{t+i}, E_t y_{t+i}), \tag{1.30}$$

where

$$\lambda(\gamma) = \left(\frac{1-\xi_p \gamma^{\theta-1}}{\xi_p \gamma^{\theta-1}}\right)(1-\xi_p \beta \gamma^{\theta}).$$

Finally, a model with roundabout production and zero-trend inflation ($0 < \phi \leq 1$ and $\gamma = 1$) delivers the NKPC:

$$\pi_t = \lambda(\phi) mc_{st} + \beta \mathrm{E}_t \{\pi_{t+1}\}, \tag{1.31}$$

where

$$\lambda(\phi) = \frac{(1-\xi_p)(1-\xi_p \beta)}{\xi_p}(1-\phi).$$

The above expressions establish that the slope coefficients of (1.29) and (1.31) on the one hand, and the slope coefficients of (1.30) and (1.28) on the other hand, are proportional, with the factor of proportionality being measured by $(1-\phi)$. The NKPC-slope coefficients decrease with any increase in either γ or ϕ.

1.5 Calibration and Results

1.5.1 Calibrated Parameters

We need to assign values to the following parameters: the subjective discount factor β, the technology parameters ϕ and α, the elasticity of substitution between differentiated goods θ, and the probability of price non-reoptimization ξ_p. The values assigned to these parameters are summarized in Table 1.1.

The subjective discount factor is $\beta = (0.965)^{1/4}$. The elasticity of substitution between differentiated goods θ determines the steady-state markup of prices over marginal cost, with the markup given by $\mu_p = \theta/(\theta-1)$. Studies by Basu and Fernald (1997, 2000) suggest that when controlling for factor capacity utilization rates, the value-added markup is about 1.05. Without any utilization correction, the value-added markup would be more in the range of 1.12. Rotemberg and Woodford (1997) suggest a higher value-added markup of about 1.2 without correcting for factor utilization. Since we do not focus on variations in factor utilization, we set $\theta = 10$, so the value-added markup is 1.11. The steady-state ratio of the fixed cost to gross output F/X is set accordingly, so that the steady-state profits for firms are zero (and there will be no incentive to enter or exit the industry in the long run). With zero economic profit, the parameter α corresponds to the share of payments to capital in total value-added in the National Income and Product Account (NIPA) and is about 0.4 (see also Cooley and Prescott, 1995)

The parameter ϕ measures the share of payments to intermediate input in total production cost or cost share. With markup pricing, it equals the product of the steady-state markup and the share of intermediate input in gross output or revenue share. We rely on two different sources of data to calibrate ϕ for the postwar U.S. economy. The first source is a study by Jorgenson, Gollop and Fraumeni (1987) suggesting that the revenue share of intermediate input in total manufacturing output is about 50 percent. With a steady-state markup of 1.11, this implies $\phi = 0.56$. The second source relies on the 1997 Benchmark Input-Output Tables of the Bureau of Economic Analysis (BEA, 1997). In the Input-Output Table, the ratio of "total intermediate" to "total industry output" in the manufacturing sector or revenue share is 0.68, implying $\phi = 0.745$. Hence, according to our two alternative sources of data, admissible values of ϕ range from 0.56 to 0.745. Bergin and Feenstra (2000) assume even higher values of ϕ, from 0.8 to 0.9, which appears excessively high based on our calculations. Huang, Liu and Phaneuf (2004) and Nakamura and Steinsson (2010) choose $\phi = 0.7$. We take a more conservative stand and set the baseline value of ϕ at 0.6. Later, we assess the sensitivity of our findings to higher values of ϕ.

The parameter ξ_p, which measures the probability of price non-reoptimization, is fixed as follows. In a survey of postwar evidence on U.S. price behavior, Taylor (1999) documents that prices have changed about once a year on average. Using summary statistics from the Consumer Price Index micro data compiled by the

U.S. Bureau of Labor Statistics for 350 categories of consumer goods and services, Bils and Klenow (2004) document that the median waiting time between price adjustments has been 4.3 months when price adjustments occuring during temporary sales are taken into account, while it has been 5.5 months when they are not. Their evidence, however, covers only a very short period of time, the years 1995-1997. Using a fewer categories of consumer goods and services, they report evidence suggesting that for the longer period 1959-2000 the frequency of price adjustments is much lower than for the years 1995-1997. Nakamura and Steinsson (2008) provide estimates of the frequency of price changes ranging from 8 to 11 months when product substitutions and temporary sales are both excluded, and from 7 and 9 months when only temporary sales are excluded.

In light of these studies, we set the baseline value of ξ_p at 3/4 (see also Ascari, 2004; Ascari and Ropele 2007). Bils and Klenow (2004) emphasize the median as their measure of waiting time between price adjustments. Approximating the waiting time to the next price change by ξ_p^t, the median waiting time between price changes is given by $-\ln(2)/\ln(\xi_p)$.[7] Setting $\xi_p = 3/4$ implies that the median waiting time between price changes is 7.2 months, which is in the range of admissible values from micro level evidence. We later assess the sensitivity of our findings to lowering ξ_p.

1.5.2 The NKPC: Intermediate Goods vs Trend Inflation

A key factor determining short-run inflation dynamics is the slope coefficient of the NKPC. Tables 1.2 - 1.4 provide a quantitative assessment of this coefficient in the four models described in the previous section. For ξ_p and β set at their baseline values, the slope coefficient of the sticky-price model without intermediate inputs and a log-linearization around zero steady-state inflation, λ, is 0.086. Nelson (1998) provides evidence showing that to match the high positive serial correlation of inflation found in the U.S. data, the standard Calvo

7. See Cogley and Sbordone (2008, footnote 19).

sticky-price model with zero trend inflation must assume a very high probability of price non-reoptimization, in the neighborhood of 0.9. With $\xi_p = 0.9$, the slope coefficient on marginal cost is then 0.012, or 7 times smaller than implied by our baseline calibration. The sensitivity of inflation to real marginal cost in the basic sticky-price model is too high to match inflation persistence.

Ascari (2004) assesses the sensitivity of the NKPC-slope coefficient to trend inflation in the sticky-price model. Table 1.2 reports values of $\lambda(\gamma)$ corresponding to alternative levels of trend inflation. For a trend inflation of 2% annually, the slope coefficient decreases by 30% with respect to a log-linearization around zero steady-state inflation. If annualized trend inflation is 4%, the slope coefficient is roughly cut in half, decreasing by 53% with respect to the standard model. However, to generate a slope coefficient of about 0.012, trend inflation would have to be 8%. Such a high value of trend inflation is implausible for the U.S. economy. Indeed, the average rate of U.S. inflation has been 3.57% from 1960:I to 2011:III. Once dividing the sample period into two subperiods, the average rate of inflation is 4.92% (roughly 5%) between 1960:I and 1983:IV, and 2.4% from 1984:I to 2011:III. Clearly, 8% trend inflation is too high for the NKPC with non-zero steady-state inflation to generate a slope coefficient that would be consistent with the inertial behavior of inflation. But as emphasized by Ascari (2004), the findings presented in Table 1 suggest that a log-linear approximation expressing the dynamics of inflation as a function of the future expected path of marginal costs in a zero steady-state inflation substantially deteriorates as trend inflation increases.

Next, we examine the response of inflation to real marginal costs in a sticky-price model with intermediate inputs and zero steady-state inflation. Table 1.3 reports the slope coefficient, $\lambda(\phi)$, for $\gamma = 1$ and a share of intermediate inputs ϕ ranging from 0.6 to 0.8. With $\phi = 0.6$, the slope coefficient on marginal cost is 0.034, which represents a huge drop of 60% with respect to the basic sticky-price model ($\phi = 0$ and $\gamma = 1$). Interestingly, this has more or less the same effect as assuming an annualized inflation trend of 4.75% in a model without intermediate goods. Note, however, that for a share of intermediate inputs set at

0.6, accounting for a roundabout structure with zero trend inflation will not lower the slope coefficient by enough to be consistent with inflation persistence. This would require a share of intermediate inputs of 0.8 or higher. Still, our findings suggest that adding input-output linkages between firms to a sticky-price model with zero-trend inflation has a stronger impact on inflation dynamics under a plausible share of intermediate inputs than embedding modest steady-state rates of inflation in a model without intermediate inputs.

Our last model combines sticky prices with intermediate inputs and positive trend inflation. Table 1.4 reports the slope coefficients $\lambda(\gamma, \phi)$ for alternative values of γ and ϕ. Their joint effect on the NKPC is striking. For example, with an annualized rate of trend inflation of only 1% and a share of intermediate inputs of 0.6, the NKPC-slope coefficient is quite small at 0.029, which represents a huge decline of about 66% with respect to the basic model with zero trend inflation and no intermediate inputs. More importantly, for a trend inflation rate between 3 and 5% and a share of intermediate inputs between 0.6 to 0.8 (the shaded area in Table 1.4), the slope coefficient on marginal cost varies between 0.006 and 0.02, which is broadly consistent with short-run inflation dynamics. Furthermore, even for a trend inflation rate as low as 1 or 2%, the model delivers small slope coefficients insofar as the share of intermediate inputs is high (between 0.7 and 0.8). Figure 1.1 summarizes the effect of alternative values of γ and ϕ on the NKPC-slope coefficient, $\lambda(\gamma, \phi)$.

The last question we ask is whether the percentage reductions in the NKPC-slope coefficient remain large when the frequency of price adjustments is set at a higher value. We lower ξ_p from 3/4 to 2/3, which is equivalent to decreasing the median waiting time between price adjustments from 7.2 to 5.1 months. ξ_p being lower, the slope coefficient increases. Specifically, in the basic model, the slope coefficient λ doubles when ξ_p decreases from 3/4 to 2/3 (0.086 *vs* 0.17). With such a high frequency of price adjustments, the standard Calvo-pricing model fails dramatically to capture inflation dynamics. The percentage declines in the slope coefficients corresponding to admissible values of γ and ϕ remain very large, even at low trend inflation rates. For a 3% annualized trend inflation rate, the slope coefficient decreases by 73, 79 and 86% if the share of intermediate inputs is 0.6, 0.7 and 0.8, respectively. Therefore, the impact of positive trend inflation and roundabout production on the NKPC is still very large when the frequency of price changes is very high.

1.6 Conclusion

For years, New Keynesian Phillips Curve models have assumed zero steady-state inflation (e.g., Christiano, Eichenbaum and Evans, 2005; Smets and Wouters, 2007), presumably as a matter of convenience. However, a growing body of research tends to demonstrate that this assumption can be misleading, giving a distorted picture of the sources of inflation dynamics and of the way monetary policy should be conducted (e.g., Ascari, 2004; Ascari and Ropele, 2007; Coibion and Gorodnichenko, 2011).

While recognizing the significance of accounting for positive trend inflation in new keynesian models, the present paper has emphasized another important mechanism contributing to inflation persistence: the multiplier for price stickiness. This multiplier arises from the interaction between sticky prices and a realistic roundabout production structure characterizing modern industrialised economies. Taken together, positive trend inflation and roundabout production act as powerful mechanims lowering the response of inflation to real marginal costs, and strongly affecting the New Keynesian Phillips Curve. Unifying these promising mechanisms into DSGE models with nominal rigidities and other types of frictions should come high on the agenda of future research.

Parameter	Value
Subjective discount factor	$\beta = (0.965)^{1/4}$
Elasticity of substitution between differentiated goods	$\theta = 10$
Probability of price non-reoptimization	$\xi_p = 3/4$
Share of intermediate input	$\phi = 0.6$

Table 1.1 Calibrated Parameters Values

$\lambda = 0.086$	$\lambda(\gamma)$	$(\lambda - \lambda(\gamma))/\lambda$
$\gamma = (1.01)^{1/4}$	0.073	15%
$\gamma = (1.02)^{1/4}$	0.061	30%
$\gamma = (1.04)^{1/4}$	0.04	53%
$\gamma = (1.06)^{1/4}$	0.024	72%
$\gamma = (1.08)^{1/4}$	0.012	86%
$\gamma = (1.1)^{1/4}$	0.004	95%

Table 1.2 The NKPC-Slope Coefficient With Positive Trend Inflation

$\lambda = 0.086$	$\phi = 0.6$	$\phi = 0.7$	$\phi = 0.8$
$\lambda(\phi)$	0.034	0.026	0.017
$(\lambda - \lambda(\phi))/\lambda$	60%	70%	80%

Table 1.3 The NKPC-Slope Coefficient With Intermediate Goods and Zero Trend Inflation

$\lambda = 0.086$	$\phi = 0.60$	$\phi = 0.65$	$\phi = 0.70$	$\phi = 0.75$	$\phi = 0.8$
$\gamma = (1.01)^{1/4}$	0.029	0.025	0.022	0.018	0.015
	66%	70%	75%	79%	83%
$\gamma = (1.02)^{1/4}$	0.024	0.021	0.018	0.015	0.012
	72%	75%	79%	82%	86%
$\gamma = (1.03)^{1/4}$	0.02	0.017	0.015	0.012	0.010
	77%	80%	83%	86%	88%
$\gamma = (1.04)^{1/4}$	0.016	0.014	0.012	0.010	0.008
	81%	84%	86%	88%	91%
$\gamma = (1.05)^{1/4}$	0.013	0.011	0.009	0.008	0.006
	85%	87%	89%	91%	93%
$\gamma = (1.06)^{1/4}$	0.010	0.008	0.007	0.006	0.005
	89%	90%	92%	93%	94%
$\gamma = (1.08)^{1/4}$	0.005	0.004	0.0036	0.003	0.002
	94%	95%	96%	96%	97%
$\gamma = (1.1)^{1/4}$	0.0018	0.0015	0.0013	0.0011	0.0010
	98%	98%	98%	99%	99%

Table 1.4 The NKPC-Slope Coefficient With Intermediate Goods and Positive Trend Inflation

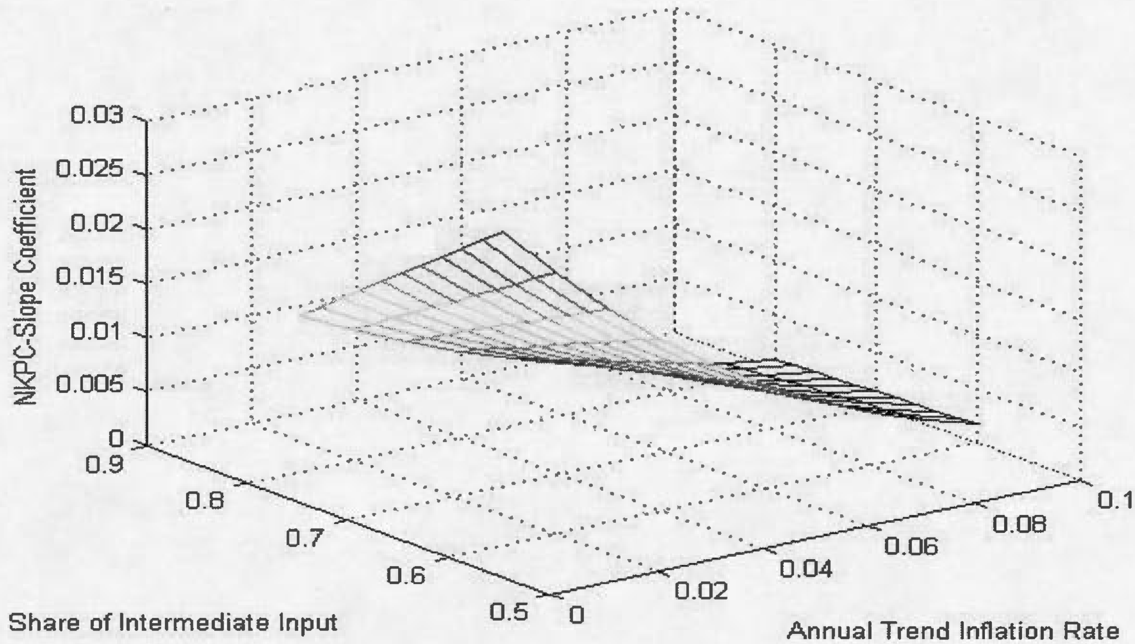

Figure 1.1 The NKPC-Slope Coefficient with Different Values of trend inflation γ and the share of intermediate goods ϕ

CHAPTER II

INFLATION PERSISTENCE IN DSGE MODELS: AN EMPIRICAL ANALYSIS

Abstract

This paper simulates a DSGE model with roundabout structure of production and positive trend inflation to assess inflation persistence observed in the U.S. data. Our simulation results provide empirical evidence in favor of intermediate goods. In effect, we find that positive trend inflation appears to have a negligible impact on inflation persistence when allowing for roundabout production. Consequently, the multiplier for price stickiness, stemming from the interaction between sticky prices and intermediate goods turns out to be the key driving force behind U.S. inflation persistence.

JEL classification: E31, E32.

Keywords: Inflation persistence; roundabout production; positive trend inflation.

2.1 Introduction

Inflation persistence is considered as the long-run effect of a shock to inflation - given a shock that raises inflation today by 1%, by how much do we expect it to be higher at some future date and how long (if ever) will it take to return to its previous level (Pivetta and Reis, 2007). This property of inflation is extremely important to be fully understood especially by central banks, which are responsible for stabilizing inflation at low levels (Sbordone, 2007).

The main goal of this paper is to improve our understanding of inflation persistence in a DSGE framework. From this point of view, Ascari (2004) studies short terms dynamics of inflation through a NKPC's slope coefficient analysis, in a standard Calvo staggered price model allowing for positive steady-state inflation. He shows that positive trend inflation may significantly decrease the slope coefficient on real marginal costs.

The central critic of dealing with inflation persistence based on the NKPC-slope coefficient is that, inflation dynamics are solely viewed through the variations of real marginal costs. By doing so, the impact of other components of the NKPC on inflation dynamics is ignored. Therefore, the contribution of the present paper is to overcome the shortcomings of this type of analytical approach, by proposing an alternative perspective of the study of inflation persistence. Here, the inflation persistence analysis relies on the simulation of a model economy, in order to account for the contribution of all the variables in the economy to short term dynamics of inflation.

To do so, we build on the work of Phaneuf and Tchakondo (2012). The authors extend the Ascari (2004)'s DSGE model, to study U.S. inflation persistence in a framework where for the first time, they take in account roundabout production structure and positive trend inflation. They find, in line with Ascari (2004) that, trend inflation plays a non negligible role in explaining short term inflation dynamics. They also show that, positive trend inflation and intermediate goods, taken together, act as powerful mechanisms lowering the response of inflation to real marginal costs, and strongly affecting the NKPC. Moreover, Phaneuf and Tchakondo (2012) highlights the multiplier for price stickiness, arising from the interaction between sticky prices and intermediate goods as a key source of inflation persistence.

However, contrary to Ascari (2004), we also embed here various frictions such as habit formation, costs of adjustment in capital accumulation and variable capital utilization. The rationale behind this is that, these frictions have become quite standard in the DSGE literature, and seem to be unavoidable to capture the empirical persistence in the macroeconomic data (CEE, 2005; Smets and Wouters, 2003). In fact, the habit formation in consumption is used to reveal the necessary empirical persistence in the consumption process (see, e.g., Bouakez, Cardia and Ruge-Murcia, 2005; Smets and Wouters, 2003; Fuhrer, 2000; and McCallum and Nelson, 1999). The capital adjustments costs do play a critical role in accounting for the dynamics of investment, since they induce inertia in investment, causing it to adjust slowly to shocks (see, e.g., Groth and Khan, 2006; CEE, 2005; and Smets and Wouters, 2003). Finally, the variable capital utilization rate aims at smoothing the adjustment of the rental rate of capital in reaction to changes in output (see, e.g., Khan and Tsoukalas, 2012; CEE, 2005; Smets and Wouters, 2003; King and Rebelo, 2000).

Then, from the overall model, we simulate five specific types of models. The first one is the model with sticky prices, zero-trend inflation without intermediate goods and real frictions. We refer to this model as the basic Calvo model or SP model with SP standing for sticky prices. Adding the real frictions mentioned earlier to the basic Calvo model gives rise to the SP-RF model where RF stands for real frictions. Furthermore, the third model is the SP-RF model with positive trend inflation without intermediate goods, called the SP-RF-PT model with PT indicating positive trend. The fourth model is the SP-RF model with intermediate goods but without positive trend inflation, say, the SP-RF-RP model where RP means roundabout production. Finally, the SP-RF model with intermediate goods and positive trend inflation, the SP-RF-RP-PT model, is considered as our benchmark model.

In addition, we generate from each specific simulated model the autocorrelations coefficients of inflation. These theoretical autocorrelations coefficients of inflation are in turn compared with those suggested by U.S. data. We show that, in order for the SP model to replicate the data, the probability of price non-reoptimization must be set at a high level around 0.96, implying an average waiting time between price adjustments of 6.25 years. For the SP-RF model, the probability must be close to 0.9, suggesting an average waiting time between price adjustments of 2.5 years or more, which is clearly counterfactual. As a consequence, the basic Calvo model even improved, is hardly reconcilable with inflation persistence unless assuming an implausibly low frequency of price adjustments.

With regard to the The SP-RF-PT model, we find that trend inflation must almost reach 11% to account for inflation persistence observed in U.S. data. Such a high value of trend inflation is unlikely for the U.S. economy. The SP-RF-RP model suggests a share of intermediate goods above 0.7 to be consistent with inflation persistence. For the SP-RF-RP-PT model, our findings suggest that, for realistic levels of trend inflation, intermediate goods supersede positive trend inflation in accounting for inflation persistence. For instance, with a share of intermediate goods set at 0.7, the autocorrelation coefficients of inflation are decreasing with positive trend inflations ranging from 1% to 5%, and are all lesser than those obtained with zero-trend inflation. Therefore, positive trend inflation appears to have a negligible impact on inflation persistence when allowing for intermediate goods. In other words, the multiplier for price stickiness, stemming from the interaction between sticky prices and intermediate goods turns out to be the key driving force behind inflation persistence.

Finally, when the frequency of price adjustments is high with a probability of price non-reoptimization set at 2/3, our results also highlight the negligible contribution of trend inflation to inflation dynamics when taking in account intermediate goods. So, the scope of positive trend inflation stressed in the literature as in Ascari (2004), appears to be overestimated and exaggerated in the presence of roundabout structure of production.

The rest of the paper is organized as follows. Section 2 describes the model economy. Section 3 discusses calibration issues. Section 4 presents the empirical evidence regarding inflation persistence in the U.S. economy and our main results. Section 5 concludes.

2.2 The Model Economy

The model economy consists of households, a representative final good producer, a continuum of intermediate goods producers indexed by $j \in [0, 1]$ and a government conducting monetary policy.

2.2.1 Households

Households consume goods and services, supply a labor to labor market, rent capital services to firms, and make investment and capital utilization decisions. There are costs related to adjusting the flow of investment and capital utilization decisions.

The utility function is the same as Chari, Kehoe and McGrattan (2010) but features the habit formation as in Christiano, Eichenbaum and Evans (2005). So, the preferences of the representative household are given by

$$E_t \sum_{t=0}^{\infty} \beta^t \left\{ \left[b(C_t - hC_{t-1})^{(\eta-1)/\eta} + (1-b)\left(M_t/P_t\right)^{(\eta-1)/\eta} \right]^{\eta/(\eta-1)} (1-L_t)^e \right\}^{1-\chi} /(1-\chi), \tag{2.1}$$

where $\beta \in (0, 1)$ denotes the subjective discount factor, C_t is consumption of final good, M_t is nominal stock of money, P_t is the price of final good, M_t/P_t is real money balances, L_t is labor. E_t indicates the conditional expectation operator, b is the utility weight of consumption, h is the habit formation parameter in consumption preferences, η is interest elasticity, e is the weight on leisure, and χ denotes a risk aversion coefficient.

The household's budget constraint expressed in nominal terms is

$$P_t\left[C_t + I_t + a(Z_t)K_t\right] + M_t + E_t D_{t,t+1} B_{t+1} \leq W_t L_t + R_t^k \widehat{K}_t + M_{t-1} + \widetilde{\Pi}_t + B_t + T_t, \tag{2.2}$$

where I_t is time t purchases of investment goods, B_{t+1} denotes the household holdings of a nominal bond representing a claim to one dollar in $t+1$ and costing $D_{t,t+1}$ dollars at time t, T_t indicates nominal lump-sum taxes, W_t is the nominal wage of labor, R_t^k is the nominal rental rate on capital services, $\widetilde{\Pi}_t$ denotes the nominal dividends received for the ownership of firms.

Households rent capital services to firms, and capital services $\widehat{K}_t$ are related to the physical stock of capital, K_t as follows:

$$\widehat{K}_t = Z_t K_t, \tag{2.3}$$

where Z_t is the utilization rate of capital, which is assumed to be set by the household. In (7), $R_t^k \widehat{K}_t$ denotes the household's earnings from supplying capital services. $a(Z_t)$ is an increasing convex function and represents the costs, in units of consumption goods, of setting the utilization rate to Z_t. In the steady state, $Z_t = 1$, with $a(1) = 0$ and $\psi = a''(1)/a'(1)$ is the capital utilization elasticity.

The household's stock of physical capital evolves according to the equation

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{2.4}$$

where δ is the physical capital depreciation rate. The term $S\left(\frac{I_t}{I_{t-1}}\right)$ is a convex investment adjustment cost function. It is assumed that in the steady state $S(1) = S'(1) = 0$ and $\kappa = S''(1) > 0$ indicates the investment adjustment cost parameter.

2.2.2 Firms

Let $X_t(j)$ be the quantity of differentiated goods produced by firms j for $j \in [0,1]$, and let $P_t(j)$ be the nominal prices. The aggregate production, X_t, is a composite of differentiated goods, $X_t(j)$, and produced in a perfectly competitive sector such that:

$$X_t = [\int_0^1 X_t(j)^{(\theta-1)/\theta} dj]^{\theta/(\theta-1)}, \tag{2.5}$$

where $\theta \in (1, \infty)$ is the elasticity of substitution between the goods. The corresponding price index, P_t, is the price of the composite good related to the prices, $P_t(j)$, and expressed as:

$$P_t = [\int_0^1 P_t(j)^{(1-\theta)} dj]^{1/(1-\theta)}. \tag{2.6}$$

The demand function for good of type j resulting from optimizing behavior in the aggregation sector is given by

$$X_t^d(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\theta} X_t. \tag{2.7}$$

The production of good j requires the use of capital, labor, and intermediate inputs:

$$X_t(j) = \Gamma_t(j)^{\phi}[\widehat{K}_t(j)^{\alpha} L_t(j)^{1-\alpha}]^{1-\phi} - F, \tag{2.8}$$

where $\Gamma_t(j)$ is the input of intermediate goods, $\widehat{K}_t(j)$ is the input of capital services, $L_t(j)$ denotes total hours worked, and F is a fixed cost which is identical across firms. The parameter $\phi \in (0,1)$ measures the elasticity of output with respect to intermediate input, and the parameters $\alpha \in (0,1)$ and $(1-\alpha)$ are the elasticities of value-added with respect to the capital and labor input, respectively.

Each firm acts as a price-taker in the input markets and as a monopolistic competitor in the product market. A firm chooses the price of its product, taking the demand schedule in (2.7) as given. Prices are set according to the mechanism spelled out in Calvo (1983). In each period, a firm faces a constant probability $1-\xi_p$ of reoptimizing its price, with the ability to reoptimize being independent across firms and time.

A firm j allowed to reset its price at date t chooses a price $P_t(j)$ that maximizes its profits,

$$E_t \sum_{\tau=t}^{\infty} (\xi_p)^{\tau-t} D_{t,\tau}[P_t(j)X_\tau^d(j) - V(X_\tau^d(j))], \tag{2.9}$$

where E is an expectations operator, $D_{t,\tau}$ is the price of a dollar at time τ in units of dollars at time t and $V(X_\tau^d(j))$ is the cost of producing $X_\tau^d(j)$, equal to $V_\tau[X_\tau^d(j) + F]$, and V_τ denotes the marginal cost of production at time τ.

Solving the profit-maximization problem yields the following optimal pricing decision rule:

$$P_t(j) = \left(\frac{\theta}{\theta-1}\right)\left[\frac{E_t \sum\limits_{\tau=t}^{\infty} (\xi_p)^{\tau-t} D_{t,\tau} X_\tau^d(j) V_\tau}{E_t \sum\limits_{\tau=t}^{\infty} (\xi_p)^{\tau-t} D_{t,\tau} X_\tau^d(j)}\right], \tag{2.10}$$

which says the optimal price is a constant markup over a weighted average of marginal costs during the periods the price will remain effective.

Solving the firm's cost minimization problem yields the following nominal marginal cost function:

$$V_\tau = \Phi[\alpha^{-\alpha}(1-\alpha)^{\alpha-1}]^{1-\phi} P_\tau^{\phi}[(R_\tau^k)^\alpha W_\tau)^{1-\alpha}]^{1-\phi}, \tag{2.11}$$

where $\Phi = \phi^{-\phi}(1-\phi)^{\phi-1}$, R_τ^k is the nominal rental rate on capital, and W_τ is the aggregate nominal wage rate. The real marginal cost is therefore:

$$MC_{r\tau} = \left(\frac{V_\tau}{P_\tau}\right) = \Phi[\alpha^{-\alpha}(1-\alpha)^{\alpha-1}]^{1-\phi}\left[\left(r_\tau^k\right)^\alpha (w_\tau)^{1-\alpha}\right]^{1-\phi}, \tag{2.12}$$

with $r_\tau^k = R_\tau^k/P_\tau$ and $w_\tau = W_\tau/P_\tau$. The latter equation can be rewritten as:

$$MC_{r\tau} = \Phi MC_{s\tau}^{1-\phi}, \tag{2.13}$$

where

$$MC_{s\tau} = \alpha^{-\alpha}(1-\alpha)^{\alpha-1}\left(r_\tau^k\right)^\alpha (w_\tau)^{1-\alpha}, \tag{2.14}$$

denotes the real marginal cost where we do not take in account intermediate goods ($\phi = 0$). Consequently, the optimal pricing equation (2.10) becomes

$$P_t(j) = \left(\frac{\theta}{\theta-1}\right)\left[\frac{E_t \sum_{\tau=t}^{\infty}(\xi_p)^{\tau-t} D_{t,\tau} P_t^\theta X_\tau (\Pi_{t+1} \times \Pi_{t+2} \times ... \times \Pi_\tau)^\theta \Phi MC_{s\tau}^{1-\phi}}{E_t \sum_{\tau=t}^{\infty}(\xi_p)^{\tau-t} D_{t,\tau} P_t^{\theta-1} X_\tau (\Pi_{t+1} \times \Pi_{t+2} \times ... \times \Pi_\tau)^{\theta-1}}\right], \tag{2.15}$$

where $\Pi_\tau = P_\tau/P_{\tau-1} = \gamma$ for $\tau = t+1, t+2, ..., t+\infty$, with Π_τ the gross inflation rate at time τ, and $\gamma \geq 1$ reflecting trend inflation. Following Ascari (2004), Ascari and Ropele (2007), and Phaneuf and Tchakondo (2012) we allow for positive trend inflation to reflect the fact that inflation has been non zero on average during the postwar period.
Here, we expressed the trend inflation in quarterly gross values as $\gamma = (1+x\%)^{1/4}$ where $x\%$ can be interpreted as the net rate of trend inflation. So, $x\% = 0\%$ means a zero trend inflation and corresponds to $\gamma = 1$. In the same way, a positive trend inflation requires $\gamma > 1$ or equivalently $x\% > 0\%$. So, the analysis is made based on γ or $x\%$.

Moreover, given that the fraction ξ_p of firms do not reoptimize their price in period t, the aggregate price evolves according to

$$P_t = \left[\xi_p P_{t-1}^{1-\theta} + (1-\xi_p)(P_t(j))^{1-\theta}\right]^{1/(1-\theta)}. \tag{2.16}$$

Thus, log-linearizing the equations (2.15) and (2.16) around a steady state with a positive trend inflation, leads to the following generalized NKPC:

$$\pi_t = \left(\frac{1-\xi_p\gamma^{\theta-1}}{\xi_p\gamma^{\theta-1}}\right)(1-\xi_p\beta\gamma^{\theta})(1-\phi)mc_{s\tau} + \beta E_t\pi_{t+1} + (1-\gamma)F(E_t\pi_{t+i}, E_t y_{t+i}), \tag{2.17}$$

where

$$\begin{aligned} F(E_t\pi_{t+i}, E_t y_{t+i}) &= \beta(1-\xi_p\gamma^{\theta-1})\{y_t - \left(\theta + \frac{\xi_p\gamma^{\theta-1}}{1-\xi_p\gamma^{\theta-1}}\right)E_t\pi_{t+1} \\ &\quad -(1-\xi_p\beta\gamma^{\theta-1})E_t\sum_{\tau=t}^{\infty}(\xi_p\beta\gamma^{\theta-1})^{\tau-t}[(\theta-1)\pi_{t+1,\tau+1} + y_{\tau+1}]\}. \end{aligned} \tag{2.18}$$

It follows that the general formulation of the NKPC in (2.17), nests some specific well-known models in the literature. For intance, one can obtain the basic Calvo model with sticky prices which results into the basic NKPC, by abstracting from intermediate goods and assuming zero steady-state inflation ($\phi = 0$ and $\gamma = 1$). Moreover, a model without intermediate goods ($\phi = 0$), but including positive trend inflation ($\gamma > 1$), yields the NKPC introduced by Ascari (2004), and Ascari and Ropele (2007).

Finally, the conditional demand functions for the intermediate input and for the primary factor inputs used in the production of $X_\tau^d(j)$ which are derived from cost-minimization are

$$\Gamma_\tau(j) = \phi\frac{V_\tau[X_\tau^d(j) + F]}{P_\tau}, \tag{2.19}$$

$$\widehat{K}_\tau(j) = \alpha(1-\phi)\frac{V_\tau[X_\tau^d(j) + F]}{R_\tau^k}, \tag{2.20}$$

and

$$L_\tau(j) = (1-\alpha)(1-\phi)\frac{V_\tau[X_\tau^d(j) + F]}{W_\tau}. \tag{2.21}$$

2.2.3 Monetary Policy

The government injects money into the economy through nominal transfers, so $T_t = M_t^s - M_{t-1}^s$ where M^s is the aggregate nominal money supply. Furthermore, following Ascari (2004) and Ascari and Ropele (2007), we assume that steady-state money supply evolves according to the fixed rule: $M_t^s = \gamma M_{t-1}^s$, where γ is the gross steady-state growth rate of nominal money supply and the source of positive trend inflation.

2.2.4 Equilibrium and Market-Clearing Conditions

An equilibrium consists of allocations $\Gamma_t(j)$, $\widehat{K}_t(j)$, $L_t(j)$ and price $P_t(j)$ for firm j, for all $j \in [0,1]$, together with prices $D_{t,t+1}$, P_t, R_t^k, and W_t, satisfying the following conditions: (i) taking the nominal wage rate and all prices but its own as given, each firm's allocations and price solve its maximization problem; (ii) markets for bonds, capital, labor and the composite good clear; (iii) monetary policy is as specified.

Along with (2.19), the market-clearing condition for the composite good

$$Y_t + \int_0^1 \Gamma_t(j)dj = X_t,$$

implies that equilibrium real GDP is related to gross output by

$$Y_t = X_t - \phi\frac{V_t}{P_t}\left[G_t X_t + F\right], \tag{2.22}$$

where $G_t \equiv \int_0^1 [P_t(j)/P_t]^{-\theta} dj$ captures the price-dispersion effect of staggered price contracts.

Meanwhile, the market-clearing conditions $\int_0^1 \widehat{K}_t^d(j)dj = \widehat{K}_t$ for capital services, and $\int_0^1 L_t^d(j)dj = L_t^d = L_t^s$ for labor, along with (2.20) and (2.21), imply that the equilibrium aggregate capital services and labor are related to gross output by

$$\widehat{K}_{t-1} = \alpha(1-\phi)\frac{V_t}{R_t^k}[G_t X_t + F], \tag{2.23}$$

$$L_t = (1-\alpha)(1-\phi)\frac{V_t}{W_t}[G_t X_t + F]. \tag{2.24}$$

Equations (2.22), (2.23), and (2.24), together with the price-setting equation (2.10), characterize an equilibrium.

The overall resource constraint of the economy is given by

$$C_t + I_t + a(Z_t)K_t \leq Y_t. \tag{2.25}$$

2.3 Calibration

We need to assign values to the following parameters: the subjective discount factor β, the preference parameters b, h, η, e, χ, the technology parameters ϕ and α, the elasticity of substitution between differentiated goods θ, the capital depreciation rate δ, the capital adjustment cost parameter κ, the capital utilization elasticity ψ, and the probability of price non-reoptimization ξ_p. The values assigned to these parameters are summarized in Table 2.2.

The subjective discount factor is $\beta = (0.965)^{1/4}$. The preference parameters are taken from Chari, Kehoe and McGrattan (2000), so $\eta = 0.39$, b $= 0.94$, e $= 1.5$ and $\chi = 1$.[1] The habit formation parameter h ranges from 0.59 as in Smets and Wouters (2003) to very high values of 0.98 as reported by Bouakez, Cardia and

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1. The same values are used by Ascari.

Ruge-Murcia (2005). Ravn, Schmitt-Grohe, and Uribe (2006) and Ravn, Schmitt-Grohé, Uribe and Uusakala (2008) give a value of 0.86 to 0.85 respectively. We set this value to 0.8 as in Fuhrer (2000) and Boldrin, Christiano and Fisher (2001).

The elasticity of substitution between differentiated goods θ determines the steady-state markup of prices over marginal cost, with the markup given by $\mu_p = \theta/(\theta - 1)$. As in Chari, Kehoe and McGrattan (2000) and Ascari (2004) θ is set to 10, so the value-added markup is 1.11. The steady-state ratio of the fixed cost to gross output F/X is set accordingly, so that the steady-state profits for firms are zero (and there will be no incentive to enter or exit the industry in the long run). The elasticity of value-added with respect to capital input α is 0.33, and the capital depreciation rate $\delta = 1 - (0.92)^{1/4}$.

The capital adjustment cost parameter κ captures the costs associated with changing the level of capital. We know from Groth (2006) that this parameter could range from 0 to infinity. For instance, Woodford (2003) proposes a value of 3. Eichenbaum and Fisher (2004) find that for κ equal to or above 3, the implied price duration is less than one year, thus consistent with evidence from micro-studies. The same evidence is found by Gagnon and Khan (2005) for the case when κ goes to infinity. Conversely, when κ is below 3, estimates of the NKPC are less able to match micro data on the duration of prices. Furthermore, based on the results from q models literature, Groth (2006) finds a value of κ between 40 and 80 as in Eichenbaum and Fisher (2004). Meanwhile, based on the parameter estimates reported by Shapiro (1986), Groth (2006) obtains a value of κ equal to 17, and the estimates by Groth (2005) imply a value of $\kappa = 20$. Therefore, we choose $\kappa = 10$. The capital utilization elasticity ψ is fixed at 1 (Basu and Kimball, 1997; Dotsey and King, 2006).

The parameter ϕ measures the share of payments to intermediate input in total production cost or cost share. With markup pricing, it equals the product of the steady-state markup and the share of intermediate input in gross output or revenue share. We rely on two different sources of data to calibrate ϕ for the postwar U.S. economy. The first source is a study by Jorgenson, Gollop and Fraumeni (1987) suggesting that the revenue share of intermediate input in total manufacturing output is about 50 percent. With a steady-state markup of 1.11, this implies $\phi = 0.56$. The second source relies on the 1997 Benchmark Input-Output Tables of the Bureau of Economic Analysis (BEA, 1997). In the Input-Output Table, the ratio of "total intermediate" to "total industry output" in the manufacturing sector or revenue share is 0.68, implying $\phi = 0.75$. Hence, according to our two alternative sources of data, admissible values of ϕ range from 0.56 to 0.75. Bergin and Feenstra (2000) assume even higher values of ϕ, from 0.8 to 0.9, which appears excessively high based on our calculations. Huang, Liu and Phaneuf (2004) and Nakamura and Steinsson (2010) choose $\phi = 0.7$ as a benchmark for the postwar U.S. economy. So, we set ϕ at 0.7. However, for the sensitivity analysis we also consider other values of ϕ.

The parameter ξ_p, which measures the probability of price non-reoptimization, is fixed as follows. In a survey of postwar evidence on U.S. price behavior, Taylor (1999) documents that prices have changed about once a year on average. Using summary statistics from the Consumer Price Index micro data compiled by the U.S. Bureau of Labor Statistics for 350 categories of consumer goods and services, Bils and Klenow (2004) document that the median waiting time between price adjustments has been 4.3 months when price adjustments occuring during temporary sales are taken into account, while it has been 5.5 months when they are not. Their evidence, however, covers only a very short period of time, the years 1995-1997. Using a fewer categories of consumer goods and services, they report evidence suggesting that for the longer period 1959-2000 the frequency of price adjustments is significantly lower than for the years 1995-1997. Nakamura and Steinsson (2008) provide estimates of the frequency of price changes ranging from 8 to 11 months when product substitutions and temporary sales are both excluded, and from 7 and 9 months when only temporary sales are excluded.
Moreover, Bils and Klenow (2004) emphasize the median as their measure of waiting time between price adjustments. Approximating the waiting time to the next price change by ξ_p^t, the median waiting time between price changes is given by

$-\ln(2)/\ln(\xi_p)$.[2] Setting $\xi_p = 3/4$ implies that the median waiting time between price changes is 7.2 months, which is in the range of admissible values from micro level evidence. At macro level, Justiniano, Primeceri and Tambolotti (2011) estimate the probability of price non-reoptimization, in a DSGE model using U.S. quarterly data from 1954QIII to 2009QI. They find that ξ_p ranges from 0.757 to 0.817.

In light of these studies, we set the baseline value of ξ_p at 3/4 (see also Ascari, 2004; Ascari and Ropele 2007). We later assess the sensitivity of our findings to lowering ξ_p.

2.4 Results

First, we document in this section, for U.S. data, the main empirical fact that is the focus of this paper: inflation persistence - considered here as the high and slowly decaying positive autocorrelations of the quarterly first difference of the log GDP deflator (Nelson, 1998). The data used are the nonfarm business sector GDP deflator data (P_t), which are obtained from the Federal Reserve Bank of St. Louis' database. These data begin in 1959:1 and end in 2013:2.

Let π_t indicate inflation ($\Delta log P_t$) and $\rho_\pi(\mu)$ the μth autocorrelation of π_t. Following Nelson (1998), we generate simple autocorrelations coefficients of inflation. The values are summarized in Table 2.1. Table 2.1 shows that the first-order autocorrelation coefficient of inflation $\rho_\pi(\mu) = 0.83$. Inflation's higher-order autocorrelations are also large, remaining above 0.5 even at lag six. It follows that inflation displays considerable persistence.

Second, to asses the role and the importance of accounting for roundabout production in U.S. inflation persistence, we simulate several types of models derived from the generalized NKPC in (2.17). For each simulated model, we generate the autocorrelations coefficients of inflation. These theoretical autocorrelations coefficients of inflation are then compared with the data presented in Table 2.1.

2. See Cogley and Sbordone (2008, footnote 19).

The first simulated model is the basic Calvo model with sticky prices, zero steady-state inflation ($x\% = 0\%$) without intermediate goods ($\phi = 0$), habit formation, capital adjustment costs and utilization rate of capital. The autocorrelation coefficients of inflation of this model compared to the data are presented in Figure 2.1. The results suggest that in order for the basic Calvo model to replicate the data, the probability of price non-reoptimization must be set at a high level around 0.96, implying an average waiting time between price adjustments of 6.25 years. This is not consistent with microeconomic evidence on U.S. price dynamics documented by Bils and Klenow (2004) and Nakamura and Steinsson (2008), who suggest respectively a median waiting time between 4.3 and 5.5 months, and, 7 and 11 months for price adjustments.

The results from the simulated SP-RF model in Figure 2.2 show this probability must be close to 0.9. So, the average waiting time between price adjustments is 2.5 years or more, which clearly is counterfactual. Consequently, the basic Calvo model even improved, is hardly reconcilable with inflation persistence unless assuming an implausibly low frequency of price adjustments.

Here, we consider the standard Calvo sticky-price model with positive trend inflation without intermediate goods and the various frictions mentioned above, as in Ascari (2004). This model is referred to as the SP-PT model. Before going through the dynamics of inflation, it's important to highlight some feature of such an economy relative to the output. In particular, we assess the reaction of output to a monetary shock when varying trend inflation. Figure 2.3 shows the impulse responses of output to a 1% rate of money growth shock, for different values of trend inflation ($x\%$), the same like those obtained by Ascari (2004, fig. 5). Following the shock, the output increases on impact by almost 20% for a trend of 2.5%, 40% for a trend of 7.5% and 90% for a level of trend of 10%. This huge impact effect seems to be unreasonable and at the odds with the evidence about the impact of monetary policy on real economy.

However, this impact effect becomes much more low and realistic in the SP-RF-PT model, when allowing for the real frictions as we can observe in Figure

2.4. Output increases by only 1% for a trend of 2.5%, 1.2% for a trend of 7.5% and 2% for a trend of 10%. In addition to dramatically lowering the impact effect, our benchmark model also accounts for the delayed, hump-shaped response of output. It turns out that, tracking the dynamics of output and other macroeconomic variables like inflation in a DSGE model framework without these frictions as in Ascari (2004), could be misleading. That's the reason why, from now on, the analysis of the contribution of positive trend inflation, roundabout production and their interaction to inflation persistence, is done based on our benchmark model rather than the standard Calvo sticky-price model.

In that case, the simulation of the SP-RF-PT model gives the results presented in Figure 2.5. These results suggest that trend inflation, $x\%$, must at least reach 11% to account for inflation persistence observed in U.S. data. This seems unrealistic since the average rate of inflation experienced by the U.S. economy during the 1960s and 1970s, a time of high inflation is almost 5%. Consequently, the model with positive trend inflation matches inflation persistence unless assuming very high values of trend inflation.

Figure 2.6 shows the findings for the SP-RF-RP model suggesting a share of intermediate goods above 0.7 to be consistent with inflation persistence. In particular, for $\phi = 0.75$, the highest value of the range, the model is about to fit the data.

However, the SP-RF-RP model has a stronger effect on inflation persistence under a plausible share of intermediate inputs than the SP-RF-PT model. As presented in Figure 2.7, for a share of intermediate inputs equal to 0.6 and 0.7, the SP-RF-RP model has respectively, the same or more impact on the dynamics of inflation than the SP-RF-PT model with a positive trend inflation of 9% and 10%.

The last simulated model is the benchmark model or the SP-RF-RP-PT model. Figure 2.8 reports the autocorrelations coefficients of inflation for different values of trend inflation and a share of intermediate goods set at a low value of 0.6. Most surprisingly, the autocorrelations coefficients for the model with zero-trend inflation are exactly the same than those obtained with realistic trend inflations of 1% to 5%. But, these coefficients are slightly lower than the case where trend inflation is set at a higher level of 10%. Next, we do the same analysis with the share of intermediate goods set at 0.7, the post world war benchmark value, and the results are presented in Figure 2.9. Here, the autocorrelation coefficients of inflation are decreasing with positive trend inflations ranging from 1% to 5% and are all lesser than those obtained with zero-trend inflation.

As a consequence, for realistic levels of trend inflation, intermediate goods supersede positive trend inflation in accounting for inflation persistence. In other words, in a sticky price economy with intermediate goods and positive trend inflation, the multiplier for price stickiness dominates positive trend inflation when assessing the short term dynamics of inflation. Even though the positive effect of the interaction between both ingredients on these dynamics is highlighted in Phaneuf and Tchakondo (2012) through the NKPC-slope coefficient analysis, the simulation of the model economy reveals a negligible contribution of positive trend inflation to inflation persistence when allowing for intermediate goods. Therefore, the scope of positive trend inflation suggested in the literature as in Ascari (2004), and Phaneuf and Tchakondo (2012), appears to be overestimated and exaggerated in the presence of roundabout structure of production.

Finally, we extend the simulation analysis to the case when the frequency of price adjustments is high with the probability of price non-reoptimization set at 2/3. Our result confirms the previous finding that the basic model is far away from the reality. Even if The SP-RF model improves inflation persistence, it is not able to replicate the data. As far as the positive trend inflation is concerned, the results suggest unrealistic levels of trend inflation around 17% to match inflation persistence. Moreover, for a share of intermediate goods set at 0.7, the impact of the SP-RF-RP model on inflation persistence is not quite sufficient to fit the data. Since the probability of price non-reoptimization is low, the multiplier of price stickiness is less stronger than the previous case where $\xi_p = 3/4$. Nevertheless, the autocorrelation coefficients here are greater than those obtained when allowing for positive trend inflation. As a matter of fact, the autocorrelation coefficients of the SP-RF-RP-PT model are decreasing in different levels of trend inflation up to 10%, and are all lesser than those generated from the SP-RF-RP model.

These findings, in some way, corroborate our previous conclusions that when allowing for the intermediate goods, the effect of positive trend inflation on inflation persistence is negligible. This highlights the predominance of the multiplier for price stickiness on the positive trend inflation, in accounting for short term dynamics of inflation.

2.5 Conclusion

We have simulated a DSGE model featuring intermediate goods and positive trend inflation alongside other real frictions, in order to examine U.S. inflation persistence observed in the data. We show that the scope of positive trend inflation stressed in the literature as an importance source of inflation persistence, appears to be overestimated and exaggerated in the presence of roundabout structure of production. As a consequence, the multiplier for price stickiness, arising from the interaction between sticky prices and intermediate goods turns out to be the key source of U.S. inflation persistence.

	Value of μ					
	1	2	3	4	5	6
$\rho_\pi(\mu)$	0.83	0.80	0.75	0.66	0.59	0.54

Table 2.1 Inflation autocorrelations, U.S. data, 1959:1 - 2013:2.

Parameter	Value
Subjective discount factor	$\beta = (0.965)^{1/4}$
Interest elasticity	$\eta = 0.39$
Weight of consumption	$b = 0.94$
Weight on leisure	$e = 1.5$
Risk aversion coefficient	$\chi = 1$
Habit formation parameter	$h = 0.8$
Elasticity of substitution between differentiated goods	$\theta = 10$
Capital input share	$\alpha = 0.33$
Capital depreciation rate	$\delta = 1 - (0.92)^{1/4}$
Investment adjustment cost parameter	$\kappa = 10$
Capital utilization elasticity	$\psi = 1$
Share of intermediate input	$\phi = 0.7$
Probability of price non-reoptimization	$\xi_p = 3/4$

Table 2.2 Calibrated Parameters Values.

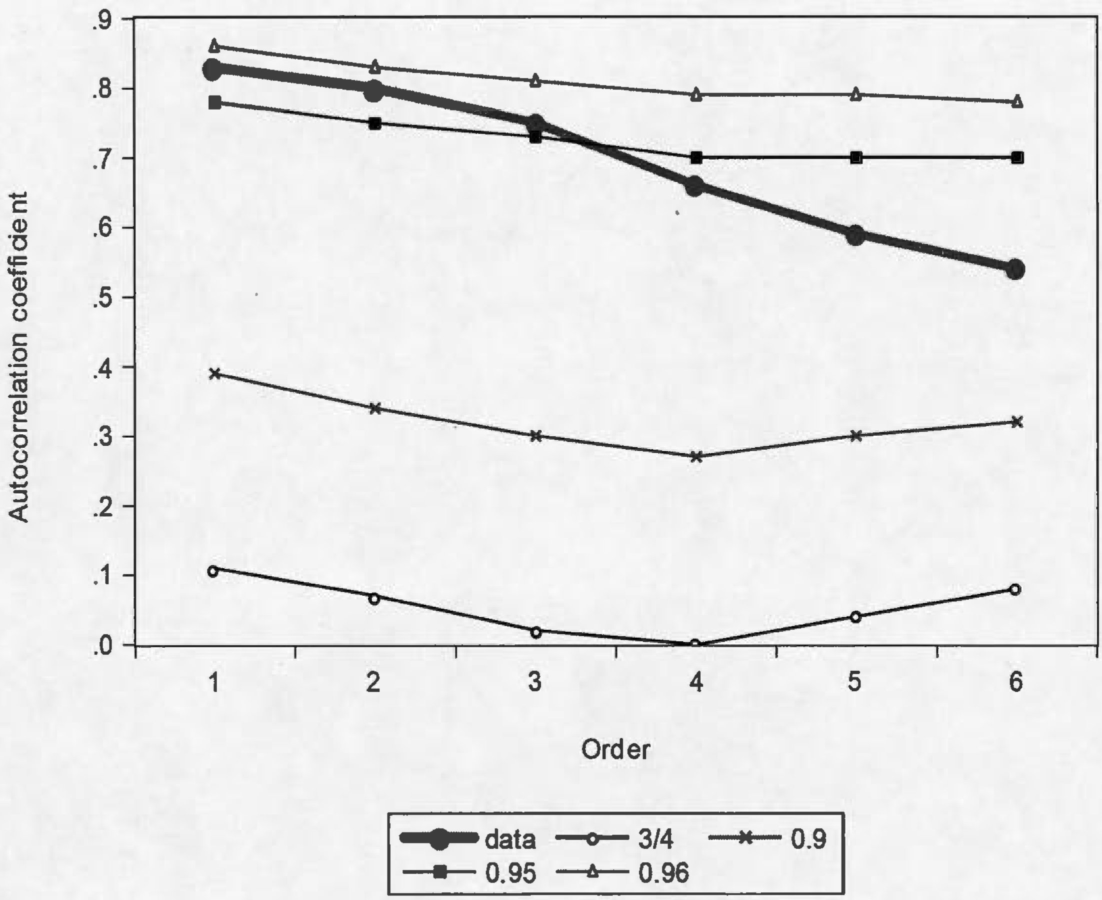

Figure 2.1 Autocorrelation Coefficients with Different Values of the Calvo probability ξ_p in the SP Model.

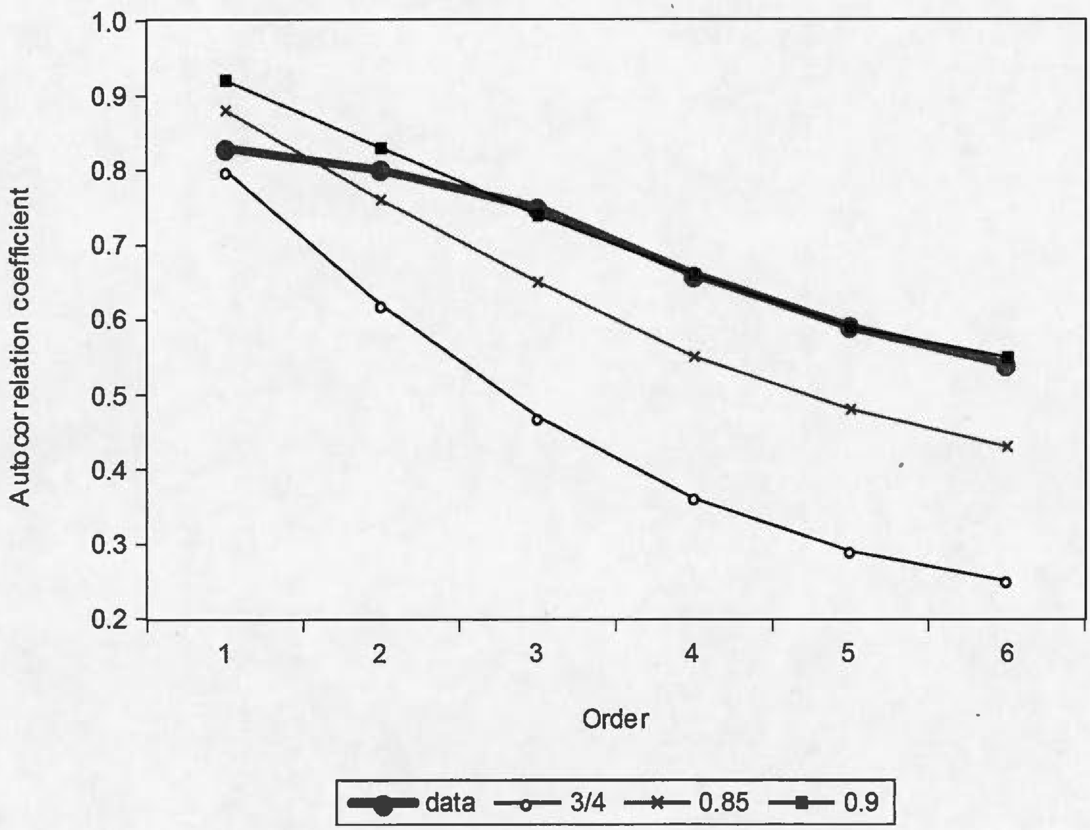

Figure 2.2 Autocorrelation Coefficients with Different Values of the Calvo probability ξ_p in the SP-RF Model.

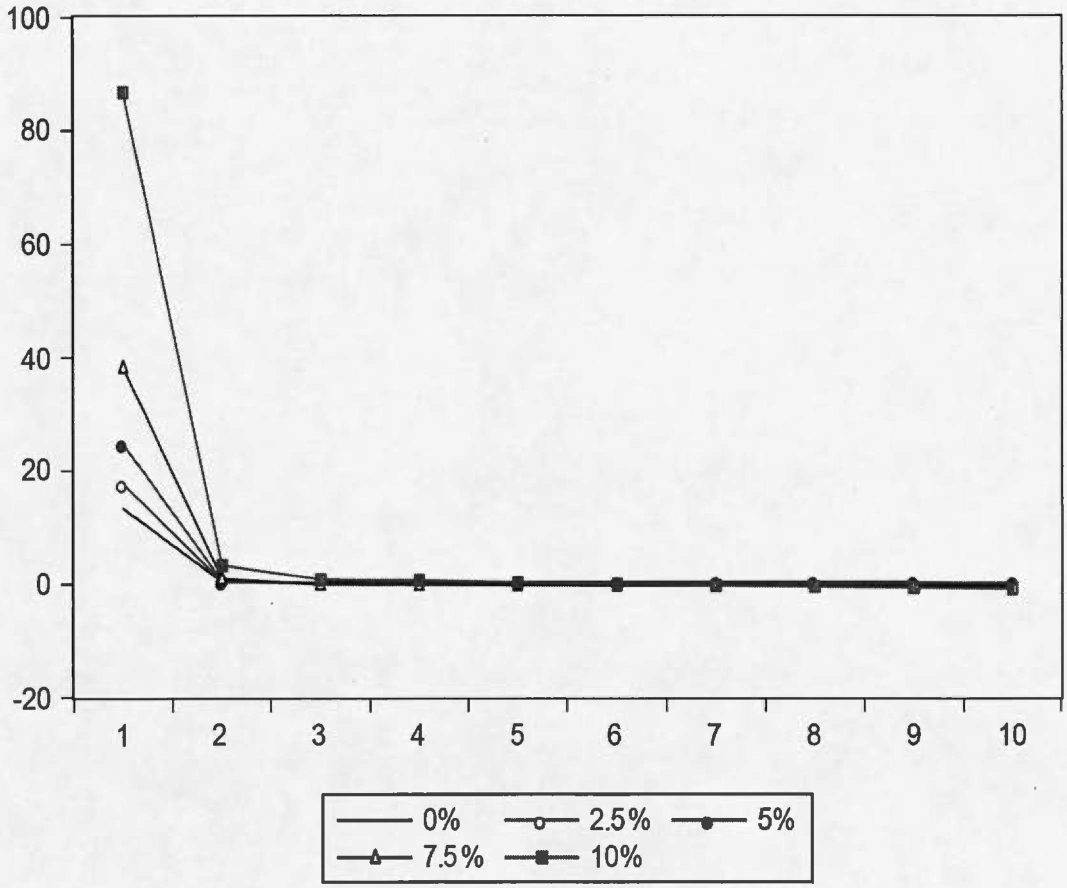

Figure 2.3 Impulse Responses of Output to a 1% Money Growth Shock with Different Values of trend inflation $x\%$ in the SP-PT Model.

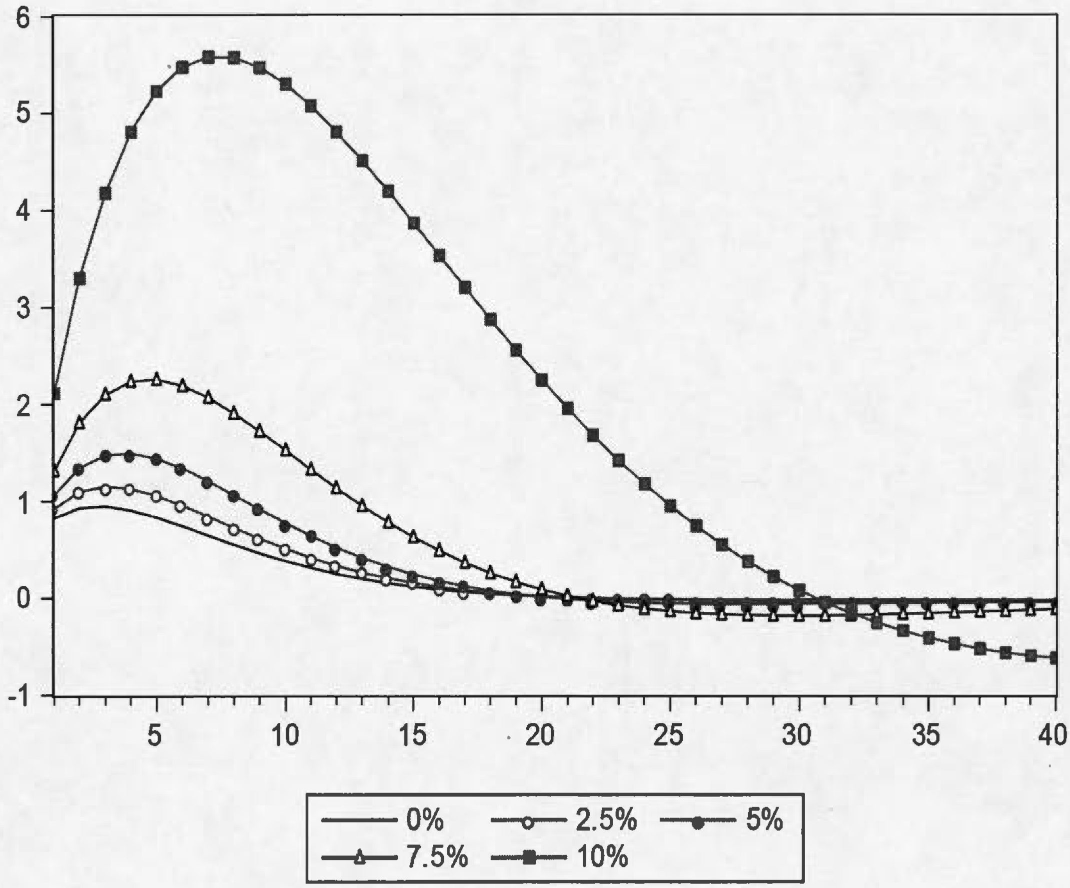

Figure 2.4 Impulse Responses of Output to a 1% Money Growth Shock with Different Values of trend inflation $x\%$ in the SP-RF-PT Model.

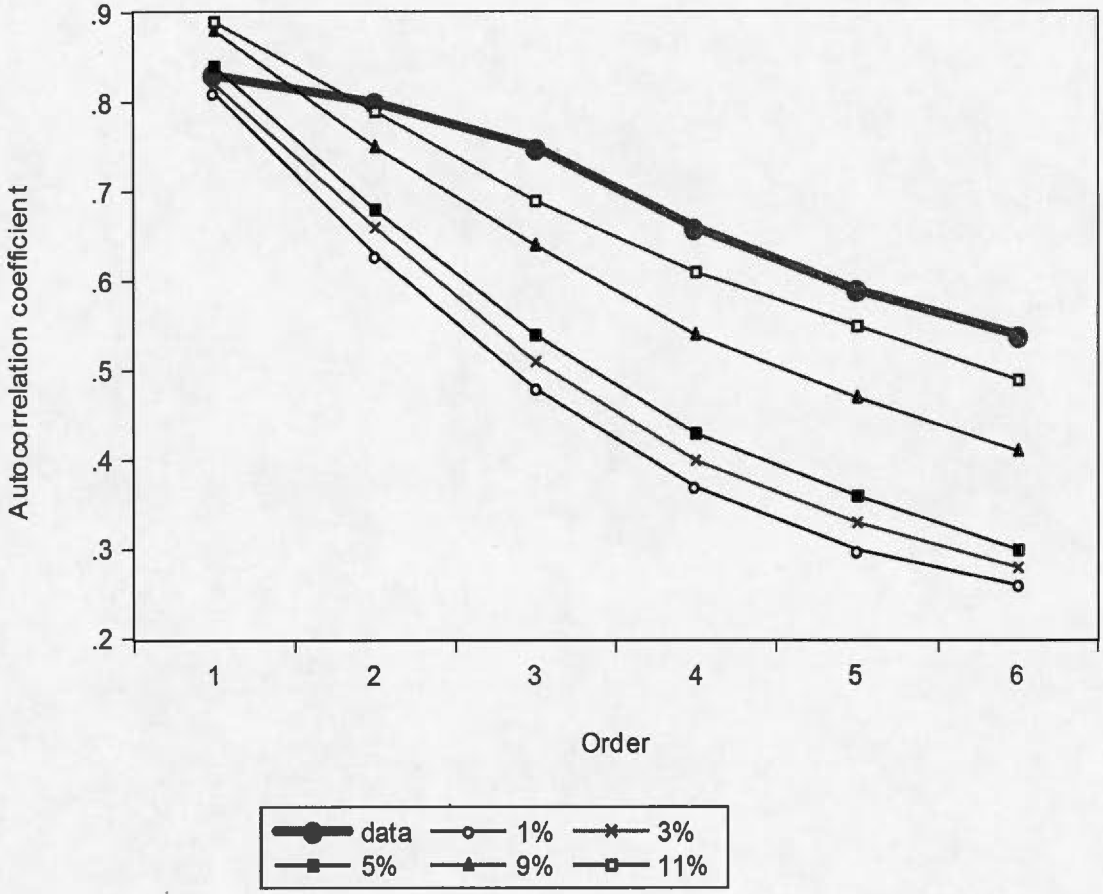

Figure 2.5 Autocorrelation Coefficients with Different Values of trend inflation $x\%$ in the SP-RF-PT Model.

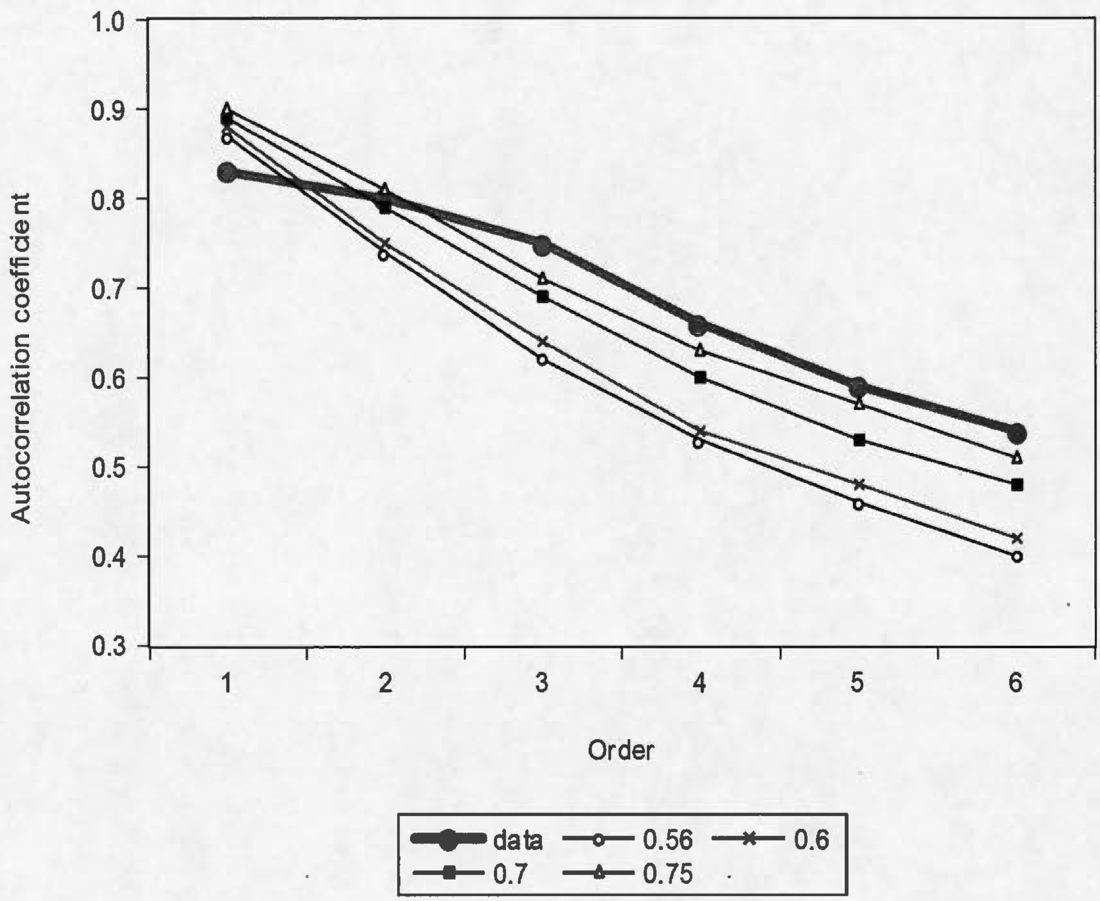

Figure 2.6 Autocorrelation Coefficients with Different Values of the share of intermediate goods ϕ in the SP-RF-RP Model.

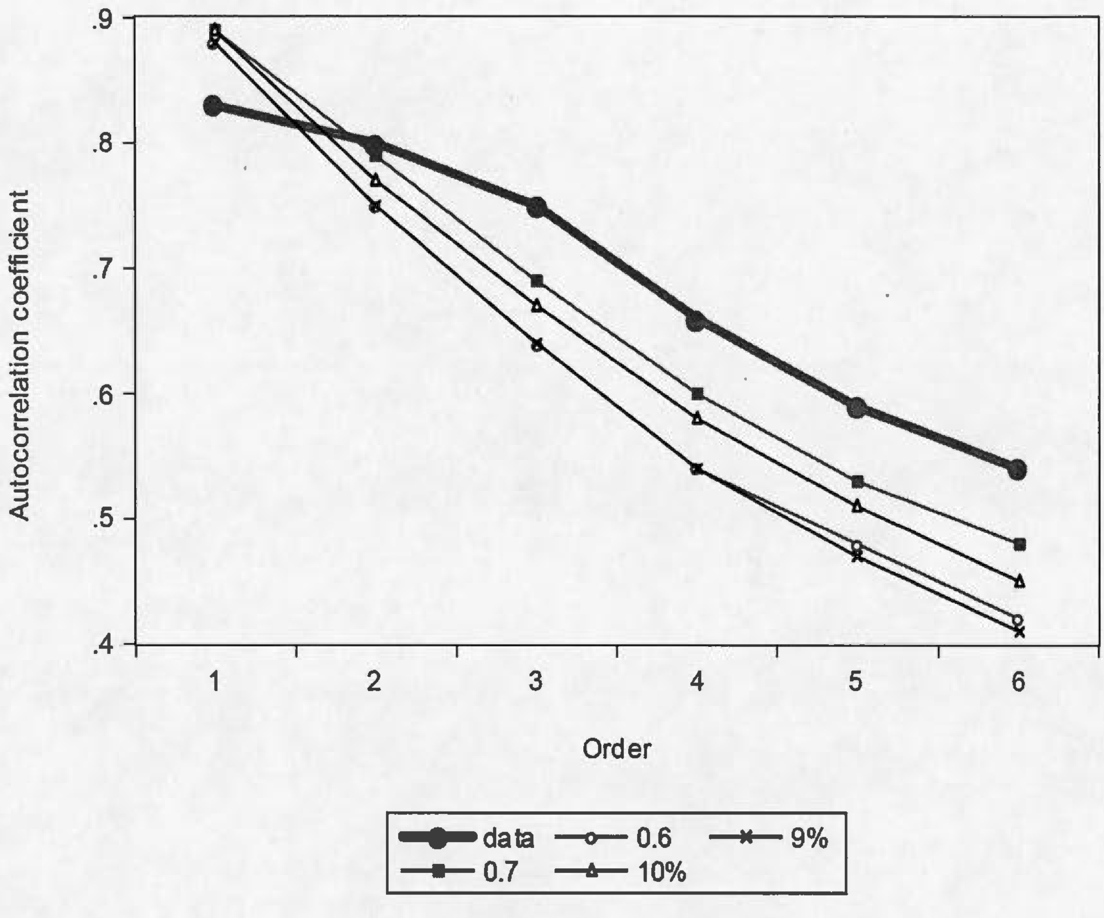

Figure 2.7 Autocorrelation Coefficients in the SP-RF-RP (▬) and SP-RF-PT (▬) Models.

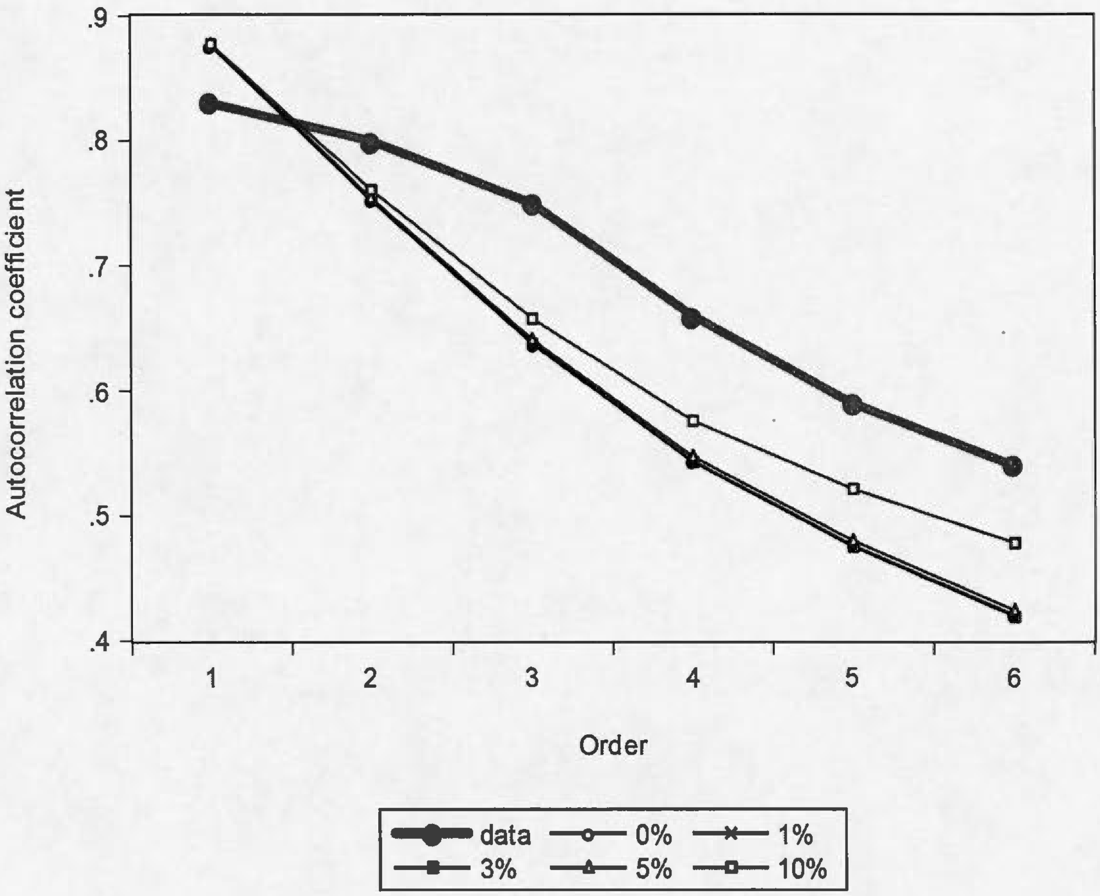

Figure 2.8 Autocorrelation Coefficients with Different Values of trend inflation $x\%$ in the SP-RF-RP-PT Model where the share of intermediate goods $\phi = 0.6$.

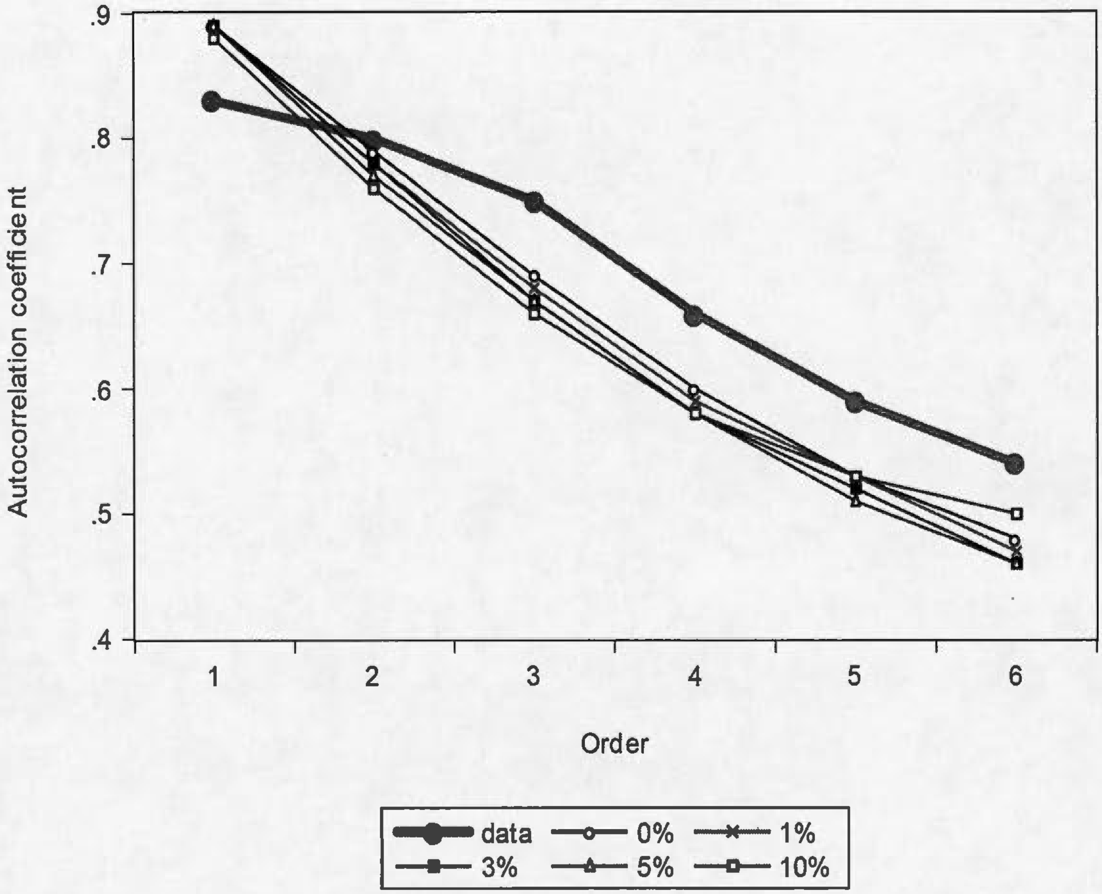

Figure 2.9 Autocorrelation Coefficients with Different Values of trend inflation $x\%$ in the SP-RF-RP-PT Model where the share of intermediate goods $\phi = 0.7$.

CHAPTER III

FINANCIAL MARKETS AND THE CAPITAL ASSET PRICING MODEL IN A DSGE FRAMEWORK

Abstract

This paper aims at better assessing the interconnections between financial markets and the real economy in a DSGE model calibrated to the U.S. economy. We incorporate a financial markets sector based on geometric Brownian motion (GBM), alongside the capital asset pricing model (CAPM) to account for households interventions on the stock market. We show that consumption, output and investment react less to a technology shock, while the nominal interest rate, inflation and labor are responding more strongly, compared to the case where financial markets are ignored. Moreover, the negative effects of a tightening monetary policy shock on output, consumption, inflation, investment and labor are more significant. Finally, a positive financial markets shock exerts a downward pressure on the nominal interest rate when the beta coefficient of the assets portfolio is positive.

JEL classification: E44, E52, G11, G12.

Keywords: CAPM; Stock markets; Monetary policy; Portfolio choice; GBM.

3.1 Introduction

The recent financial crisis has shown how difficulties in financial markets can turn into harmful consequences to the economy as a whole. It also revealed the limits of traditional monetary policy tools in a context of the zero lower bound (ZLB) on nominal interest rates (see, e.g., Coibion, Gorodnichenko and Wieland, 2012; Aruoba and Schorfheide, 2013; Barthelemy and Marx, 2013; Gavin, Keen, Richter and Throckmorton, 2013; Richter and Throckmorton, 2013). Therefore, our work seeks to deepen the analysis of the interconnections between financial markets and the real economy in DSGE models. This paper offers two main contributions. First, it provides new findings on the impacts of technology and monetary policy shocks for the economy in DSGE models where stock markets are explicitly modelled. The dynamics of financial markets are described based on geometric Brownian motion (GBM), and the capital asset pricing model (Hereafter, CAPM) that is a model of the assets portfolio choice is introduced to account for the behavior of households on the stock market in terms of purchases of risky assets. Second, it shows how a financial markets shock affects the real economy. We show that following the technology shock, the positive responses of consumption, output and investment are lower in the CAPM model than in the basic model where financial markets are ruled out. However, the decrease in the nominal interest rate, inflation and labour is more important. In fact, a positive technology shock is perceived by investors as an element that may increase the profits of firms, implying an increase in the expected return rate on risky assets. So, households invest more in risky assets instead of consuming as anticipating higher future profits of firms. This leads to a stronger decline in the nominal interest rate in the CAPM model since bonds are no longer attractive in that context. Therefore, production, investment and employment decline. Moreover, the negative effects of a tightening monetary policy shock on output, consumption, inflation, investment and labor are stronger in the CAPM model than in the basic model. A contractionary monetary policy shock causes returns on securities to fall, because financial markets anticipate a decline in the economic activity that could lower future profits of firms. This makes the decrease in such variables higher in the

CAPM model than in the basic model.

The second contribution of the paper is to assess the impact of a financial markets shock on macroeconomic variables, especially its effects on the nominal interest rate. The financial markets shock originating in the stock market is meant to represent an exogenous increase in the return on the stock market. It may result from an information in financial markets or in the economy as a whole, which is perceived by investors as something that could hit the stock market. Here, following a positive financial markets shock, returns on the stock market and on securities increase while the nominal interest rate is falling when the beta coefficient of the household assets portfolio is positive for any value of the weight invested in risky assets. Consumption, output, inflation, investment and labor increase due to the increase in expected gains made by households on financial markets. Furthermore, the positive reactions of the return on securities, consumption, output, inflation, investment and labour, and the negative response of the nominal interest rate are more important for higher positive values of the beta coefficient of the portfolio when the share of risky assets is constant. A higher value of the beta coefficient is associated with higher expected returns on the portfolio. We also find that the positive reaction of the return on securities is decreasing in the share of risky assets in the portfolio while the negative response of the nominal interest rate is increasing, for a constant beta coefficient of the portfolio. A higher value of the share of risky assets means a more diversified portfolio that is associated with low risk and return according to the CAPM framework. Thus, the increase in consumption, output, inflation, investment and labour is less large, since the anticipated profits here following the financial market shock are lower. However, the nominal interest rate reacts positively for a negative value of the beta coefficient, and then, consumption, output, inflation, investment and labour decrease.

The key implication of these results for central banking in our model economy in the case where beta coefficient of the portfolio is positive is that, an upturn in financial markets is expected to exert downward pressure on the nominal interest rate for any positive beta coefficient of the household portfolio. It turns out that, a rise in financial markets not accompanied by a reaction of the central bank should result in lower nominal interest rates. The monetary authority should then increase the nominal interest rate following the positive financial markets shock in order to bring down inflation on the one side. Conversely, a decline in financial markets gives rise to an increase in the nominal interest rate. The central bank can then lower its policy rate to boost the economy. In short, the central bank should increase the nominal interest rate when financial markets are in a strong momentum of rising, and lower it when they are experiencing sharp declines.

Finally, we find that the CAPM model is successful in reproducing most of the salient features of the U.S. economy, particularly, key macroeconomic volatilities, autocorrelations, and correlations with output.

This paper relies on Kendall and Hill (1953), Osborne (1959), Roberts (1959), Samuelson (1965), Black and Scholes (1973), Barmish and Primbs (2011) and Lochowski and Thagunna (2013) that focus on the modeling of the price and return of a stock by using GBM. But the peculiarity of the paper is that we consider here an application of GBM to the stock market index in order to highlight the movements of the stock market as a whole. Then, household portfolio choice decisions are taken into account following Markowitz (1959), Sharpe (1964), Lintner (1965), Fama (1996), and Fama and French (2004). According to the CAPM, individuals hold a portfolio that is a combination of risk-free assets (bonds) and a single risky portfolio of securities available in the stock market. "The CAPM assumes investors are risk averse and when choosing among portfolios, they only care about the mean and variance of their one-period investment return. As a result, investors choose "mean-variance-efficient" portfolios in the sense that portfolios i) minimize the variance of portfolio return given expected return and ii) maximize expected return given variance " (see, Fama and French, 2004). Thus, the CAPM enables us to highlight the trade-off between risky-free assets and risky assets in portfolio allocation decisions of individuals. This allows to make the link between financial markets and the real economy, particularly through investment decisions in stocks and bonds of the representative household. However, unlike the papers cited above, the weights to be assigned to each type of asset in the portfolio are the result of an optimization problem of the representative household even if they remain constant in the model. Furthermore, we assume that the risky portfolio is contained in the market portfolio which is defined as the portfolio representing all risky assets on the market. In other words, the risky portfolio consists of securities belonging to the market portfolio but different from the latter.

The paper is also related to some works that incorporate risky assets in DSGE models. For instance, De Paoli, Scott and Weeken (2010) investigate the behavior of asset prices using a second-order approximation to show how the risk-free real interest rate, the return on equity, the equity premium and the real and nominal term structure change with variations in some DSGE model parameters. Fornero (2011) uses a consumption capital asset pricing model (C-CAPM) and approximates it up to the third order to characterize the effects of various shocks on real interest rates, the risk premium for different bonds maturities and the term structure of interest rates. However, our paper stands out on some points. We propose a formal modelling of the stock market in a DSGE model with the CAPM to analyze especially the direct effects of a shock to the financial markets on the overall economy. In addition, assets prices and returns are modelled using GBM rather than approximation methods.

Finally, our model features a staggered price setting mechanism and habit formation as usual in DSGE models, and a positive trend inflation to reflect the fact that the central bank inflation target is different from zero (see, Ascari, 2004; Ascari and Ropele, 2007; Coibion and Gorodnichenko, 2011; Phaneuf and Tchakondo, 2012, 2013). Moreover, the model is calibrated to the U.S. economy and used to evaluate the impacts of standard supply and demand shocks and the financial markets shock on macroeconomic variables, and thus, to make explicit how financial markets are linked to the real economy.

The paper is organized as follows. Section 2 describes the model. Section 3 discusses the parameters calibration. Section 4 reports and discusses the results. Section 5 concludes.

3.2 The Model Economy

The economy is populated by a large number of firms, each producing a differentiated good indexed by $j \in [0,1]$. We allow for a stock market where households can purchase equities that are issued by firms. Finally, there is a central bank conducting an endogenous monetary policy.

3.2.1 Firms: Optimal price setting

Denote by Y_t a composite of differentiated goods $Y_t(j)$ for $j \in [0,1]$ such that $Y_t = [\int_0^1 Y_t(j)^{(\theta-1)/\theta} dj]^{\theta/(\theta-1)}$, where $\theta \in (1,\infty)$ is the elasticity of substitution between the goods. The composite good is produced in a perfectly competitive aggregate sector.

The demand functions for good of type j resulting from optimizing behavior in the aggregation sector is

$$Y_t^d(j) = \left[\frac{P_t(j)}{P_t}\right]^{-\theta} Y_t, \tag{3.1}$$

where P_t is the price of the composite good which is related to the prices $P_t(j)$ for $j \in [0,1]$ of the differentiated goods and given by $P_t = [\int_0^1 P_t(j)^{(1-\theta)} dj]^{1/(1-\theta)}$.

The production function for a good of type j is

$$Y_t(j) = A_t K_t(j)^\alpha L_t(j)^{1-\alpha}, \tag{3.2}$$

where A_t is a technology shock assumed to follow a stationary AR(1) process, $K_t(j)$ and $L_t(j)$ are the inputs of capital and labor.

Each firm acts as a price-taker in the input markets and as a monopolistic competitor in the product market. A firm can choose the price of its product, taking the demand schedule in (3.1) as given. Prices are set according to the mechanism spelled out in Calvo (1983). In each period, a firm faces a constant probability $1-\xi_p$ of reoptimizing its price, with the ability to reoptimize being independent across firms and time.

A firm j allowed to reset its price at date t chooses a price $P_t(j)$ to maximize its profits

$$E_t \sum_{\tau=t}^{\infty} (\xi_p)^{\tau-t} D_{t,\tau}[P_t(j) Y_\tau^d(j) - V(Y_\tau^d(j))], \tag{3.3}$$

where E is an expectations operator, $D_{t,\tau}$ is the price of a dollar at time τ in units of dollars at time t, and $V(Y_\tau^d(j))$ is the nominal cost of producing $Y_\tau^d(j)$, which is equal to $V_\tau Y_\tau^d(j)$, with V_τ denoting the nominal marginal cost of production at time τ.

Solving the profit-maximization problem yields the following optimal pricing decision rule:

$$P_t(j) = \left(\frac{\theta}{\theta-1}\right) \left[\frac{E_t \sum\limits_{\tau=t}^{\infty} (\xi_p)^{\tau-t} D_{t,\tau} Y_\tau^d(j) V_\tau}{E_t \sum\limits_{\tau=t}^{\infty} (\xi_p)^{\tau-t} D_{t,\tau} Y_\tau^d(j)} \right], \tag{3.4}$$

which says the optimal price is a constant markup over a weighted average of marginal costs for the periods the price will remain effective.

Solving the firm's cost minimization problem yields the following nominal marginal cost function:

$$V_\tau = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} A_\tau^{-1} \left(R_\tau^k\right)^\alpha \left(W_\tau\right)^{1-\alpha}, \tag{3.5}$$

where R_τ^k is the nominal rental rate on capital and W_τ is the aggregate nominal wage rate. The real marginal cost is therefore expressed as

$$MC_{r\tau} = \left(\frac{V_\tau}{P_\tau}\right) = \alpha^{-\alpha}(1-\alpha)^{-(1-\alpha)} A_\tau^{-1} \left(r_\tau^k\right)^\alpha \left(w_\tau\right)^{1-\alpha}, \tag{3.6}$$

with $r_\tau^k = R_\tau^k / P_\tau$ and $w_\tau = W_\tau / P_\tau$, indicating respectively the real rental rate on capital and the real wage rate. Consequently, the optimal pricing equation (3.4) becomes

$$P_t(j) = \left(\frac{\theta}{\theta-1}\right)\left[\frac{E_t \sum_{\tau=t}^{\infty}(\xi_p)^{\tau-t} D_{t,\tau} P_t^{\theta} Y_\tau (\Pi_{t+1} \times \Pi_{t+2} \times ... \times \Pi_\tau)^{\theta} MC_{r\tau}}{E_t \sum_{\tau=t}^{\infty}(\xi_p)^{\tau-t} D_{t,\tau} P_t^{\theta-1} Y_\tau (\Pi_{t+1} \times \Pi_{t+2} \times ... \times \Pi_\tau)^{\theta-1}}\right], \tag{3.7}$$

where $\gamma = \Pi_\tau = P_\tau / P_{\tau-1}$ for $\tau = t+1, t+2, ..., t+\infty$, with $\gamma \geq 1$ reflecting trend inflation, and Π_τ the gross inflation rate at time τ. Following Ascari (2004), Ascari and Ropele (2007) and Phaneuf and Tchakondo (2012, 2013) we allow for positive trend inflation to reflect the fact that the inflation target of the central bank is different from zero.

A firm that does not reset its price in a given period, nonetheless chooses the inputs of capital and labor that minimize production cost.

3.2.2 Households: Portfolio choice decisions

The representative household's preferences are described by the expected utility function:

$$E_t \sum_{t=0}^{\infty} \beta^t \left\{ \log(C_t - hC_{t-1}) - \eta \frac{L_t^{1+\chi}}{1+\chi} \right\}, \tag{3.8}$$

where $\beta \in (0,1)$ denotes the subjective discount factor, $h \in (0,1)$ is a habit formation parameter, η measures the weight on leisure in the utility function, and χ is the inverse of the Frisch wage elasticity of labour supply. C_t and C_{t-1} are current and past-period consumptions of household, and L_t is the labor.

The budget constraint in nominal terms that household faces at time t is given by the following relation:

$$P_t[C_t + I_t] + B_t + S_t^{ec} \leq W_t L_t + R_t^k K_t + R_t B_{t-1} + \frac{R_t^{S^{ec}}}{\Sigma_t} S_{t-1}^{ec} + \widetilde{\Pi}_t + T_t, \tag{3.9}$$

where, S_t^{ec} represents a set of securities or different risky assets that are especially equities issued by firms on the stock market. S_t^{ec} is assumed to be different from the market index, since we rule out the possibility for the representative household to instantly buy all the securities available on the stock market.

The household enters period t with a portfolio of total assets composed of nominal bonds B_{t-1} and nominal securities S^{ec}_{t-1}. Bonds pay the gross nominal interest rate R_t set by central bank and considered here as the risk-free rate, and securities pay the expected gross nominal interest rate $E_t(R^{S^{ec}}_{t+1})$ referred to as the expected rate of the return on securities. Σ_t is introduced to allow for risk-return differences between both types of assets in equilibrium.

Meanwhile, W_t is the nominal wage rate for labor L_t, R^k_t is the nominal rental rate on capital, $\widetilde{\Pi}_t$ is household's dividends received for the ownership of firms, and T_t indicates a lump-sum transfer the household gets from the government.

The physical capital accumulation equation is given by

$$K_{t+1} = (1-\delta)K_t + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t, \tag{3.10}$$

where δ is the physical capital depreciation rate, I_t denotes time t purchases of investment goods. The term $S\left(\frac{I_t}{I_{t-1}}\right)$ is a convex investment adjustment cost function. It is assumed that in the steady state $S(1) = S'(1) = 0$, and $\kappa = S''(1) > 0$ indicates the investment adjustment cost parameter.

The household acts as a price-taker in both goods and financial markets. The representative household chooses consumption C_t, hours worked L_t, bonds B_t, securities S^{ec}_t, the physical stock of capital for the next period K_{t+1} and investment I_t that maximize (3.8) subject to (3.9), (3.10) and a no-Ponzi-game condition.[1]

The first-order conditions for this optimization problem are :

$$\lambda_t = \beta E_t(R_{t+1}\lambda_{t+1}), \tag{3.11}$$

1. Following Alstadheim and Henderson (2006), this condition could be: $\lim\limits_{t\to\infty}\left(B_t\prod_{k=0}^{k=t-1} R_k^{-1} + S^{ec}_t\prod_{k=0}^{k=t-1}\left(R^{S^{ec}}_k\right)^{-1}\right) \geq 0$, with $\prod_{k=0}^{k=-1} R_k^{-1} \equiv 1$, and $\prod_{k=0}^{k=-1}\left(R^{S^{ec}}_k\right)^{-1} \equiv 1$.

$$\lambda_t = \beta E_t \left(\frac{R_{t+1}^{Sec} \lambda_{t+1}}{\Sigma_{t+1}} \right), \tag{3.12}$$

$$\lambda_t P_t = (C_t - hC_{t-1})^{-1} - h\beta E_t (C_{t+1} - hC_t)^{-1}, \tag{3.13}$$

$$\lambda_t P_t = \beta E_t [\lambda_{t+1} (R_{t+1}^k + (1-\delta) P_{t+1})], \tag{3.14}$$

$$\eta L_t^{\chi} = \lambda_t W_t, \tag{3.15}$$

$$P_t[C_t + K_{t+1} - (1-\delta)K_t] + B_t + S_t^{ec} = W_t L_t + R_t^k K_t + R_t B_{t-1} + \frac{R_t^{Sec}}{\Sigma_t} S_{t-1}^{ec} + \widetilde{\Pi}_t + T_t, \tag{3.16}$$

where λ_t is the Lagrangian multiplier associated with the budget constraint. From (3.11) and (3.12), we obtain in equilibrium a relation between the expected rate of return on securities ($E(R_t^{Sec})$) and the risk-free rate (R_t) that is:

$$E_t(R_{t+1}^{Sec}) = R_t E(\Sigma_{t+1}). \tag{3.17}$$

Here, Σ_t can be interpreted as a nominal gross spread rate between R_t^{Sec} and R_t expressed proportionally to the risk-free rate, and allowing for differences in terms of returns between bonds and securities.

Therefore, the weight invested by the household in risky assets proportionally to total assets $\omega_P = \frac{S_t^{ec}}{S_t^{ec}+B_t}$, and $1 - \omega_P = \frac{B_t}{S_t^{ec}+B_t}$ is the weight invested in the risk-free asset. So, from the portfolio theory[2], the expected return on the household portfolio of assets is given by

$$E_t \left(R_{P_{t+1}} \right) = \omega_P E_t \left(R_{t+1}^{Sec} \right) + (1 - \omega_P) R_t. \tag{3.18}$$

ω_P is one of the key elements of the model in the sense that it determines household portfolio choice decisions: the higher is ω_P, the more household invests in additional different risky assets[3].

2. The portfolio theory informs about investment decision making. It focus among others on the formation of an optimal portfolio of assets, particularly the determination of the best risk-return opportunities from feasible investment portfolios and the choice of the best portfolio from that feasible set (see, e.g., Bodie et al., 2005, ch.5-6).

3. The assumption that the increase in ω_P is associated with the integration of additional assets in portfolio, is necessary here to take in account the concept of portfolio diversification: the more diversified is the portfolio, the less risky it is.

3.2.3 Financial markets: The CAPM

We suppose a financial sector that is only composed of financial markets mainly a stock market, ruling out the possibility to have a banking system in the model economy. At time t, the household i can purchase securities S_t^{ec} that are firms' equities issued on the stock market.

Following Kendall and Hill (1953), Roberts (1959), Osborne (1959), Samuelson (1965), Black and Scholes (1973), Barmish and Primbs (2011) and Lochowski and Thagunna (2013), we assume that the aggregate stock market price S_t^m, which can be considered here as the stock market index is governed by the geometric Brownian motion:

$$\frac{dS_t^m}{S_t^m} = \mu dt + \sigma dBr_t, \tag{3.19}$$

where μ is a constant rate of return often called the drift and captures the annualized expected return of the stock market, σ is the volatility representing the annualized standard deviation associated with the underlying process, and Br_t is the Brownian or Wiener process.[4] This equation is viewed as a stochastic differential equation because, the stock market price S_t^m is defined implicitly by describing its changes through time random effects. Thus, the analytical solution of (3.19) is given by

$$S_t^m = S_0^m exp\left\{\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma Br_t\right\}, \tag{3.20}$$

where $S_0^m > 0$ is the initial value or the stock price at time 0.

The gross return of the stock market, $R_t^m = log\left(S_t^m / S_{t-1}^m\right)$, derived from (3.20) becomes

$$R_t^m = \left(\mu - \frac{\sigma^2}{2}\right) + \sigma\left(Br_t - Br_{t-1}\right) + \epsilon_{R_t^m}, \tag{3.21}$$

4. A Brownian motion process $Br_t, t \geq 0$, is a continuous stochastic process with the following properties: (i) it starts at zero, i.e. $Br_0 = 0$; (ii) it has independent increments; (iii) for every $t > s \geq 0, Br_t - Br_s$ has a normal distribution $N(0, t-s)$, (see, e.g., Lochowski and Thagunna, 2013; Ermogenous, 2005).

where $\epsilon_{R_t^m}$ is assumed to be a stock market shock following a stationary AR(1) process. $\epsilon_{R_t^m}$ could capture phenomena that are missing in the geometric Brownian motion (3.19) such as jumps in the stock market, particularly during a financial markets turmoil.

To see how household portfolio choice decisions and the stock market interplay, we resort to the CAPM (see, e.g., Markowitz, 1959; Sharpe, 1964; Lintner, 1965; Fama, 1996; and Fama and French, 2004) that establishes a relationship between the expected performance of the household portfolio of assets $E_t\left(R_{P_{t+1}}\right)$, and the stock market return $E_t\left(R_{t+1}^m\right)$ such that:

$$E_t\left(R_{P_{t+1}}\right) = R_t + \beta_P\left[E\left(R_{t+1}^m\right) - R_t\right], \tag{3.22}$$

where β_P called the beta coefficient of the portfolio reveals the way and the extent to which returns on the portfolio and the market move together. This CAPM equation suggests that the expected return on the household portfolio $E_t\left(R_{P_{t+1}}\right)$ is the risk-free rate R_t, plus a risk premium, which is the portfolio beta coefficient β_P, times the premium per unit of beta risk $E\left(R_{t+1}^m\right) - R_t$.

Formally, the beta of the household portfolio β_P is defined as the covariance of its return with the market return divided by the variance of the market return (see, Fama and French, 2004),

$$\beta_P = \frac{Cov\left(R_P, R^m\right)}{\sigma_{R^m}^2}. \tag{3.23}$$

So, the equation (3.22) becomes

$$E\left(R_{P_{t+1}}\right) = R_t + \frac{Cov\left(R_P, R^m\right)}{\sigma_{R^m}^2}\left[E\left(R_{t+1}^m\right) - R_t\right], \tag{3.24}$$

with the ratio $\frac{Cov(R_P,R^m)}{\sigma_{R^m}^2}$, i.e., β_P measuring the sensitivity of the return on the household portfolio to variations in the market return.

From the household portfolio decision in (3.18), the variance of the portfolio is given by $\sigma_{R_P}^2 = \omega_p^2\sigma_{R^{Sec}}^2$, where $\sigma_{R^{Sec}}^2$ indicates the mean of the variance of risky assets. This implies that (3.23) becomes

$$\beta_P = \rho_{(R_P,R^m)}\frac{\omega_P\sigma_{R^{sec}}}{\sigma_{R^m}}, \tag{3.25}$$

where $\rho_{(R_P,R^m)}$ is the correlation coefficient between returns on the portfolio and the market.
In other words, β_P is the (implicit) measure of the household portfolio risk and appears as another key element of the model, since one can infer from β_P important features of the household portfolio. In fact, the sign and magnitude of β_P depend on the type of risky assets that household wants to have in its portfolio. For instance, $\beta_P = 0$ means that the return on the household portfolio is insensitive to stock market fluctuations, implying that the portfolio of assets consists only of risk-free assets. When $\beta_P = 1$, the portfolio performance is exactly the same as the market. This suggests either that the household buys the stock market index or a portfolio that replicates perfectly the market. For $0 < \beta_P < 1$, portfolio and market returns evolve together but the portfolio weakly reacts to market fluctuations, unlike the case where the portfolio reacts strongly when $\beta_P > 1$. Finally, when $\beta_P < 0$, portfolio and market returns move in opposite directions. This can be the case when the representative household wants to take a reverse position on the market, for example via a call or a put.[5] In sum, by affecting household investment decisions, β_P and ω_P also influence household consumption choices. Therefore, they could be considered as factors linking financial markets and the real economy, and playing an important role in the transmission of shocks from financial markets to the real economy and vice versa.

Another way to analyze the interconnections between financial markets and investment and consumption decisions of the representative household can be understood from (3.18) and (3.22), by the following relation:

$$\omega_P \left[E\left(R_{t+1}^{Sec}\right) - R_t\right] = \beta_P \left[E\left(R_{t+1}^{m}\right) - R_t\right]. \tag{3.26}$$

This relation suggests that the risk premium of the portfolio is equivalent to the spread between the mean rate on securities and the risk-free rate, times the weight invested in risky assets. The distinctive characteristic of (3.26) is that it highlights a relation between financial market features (R_t^m, R_t^{Sec}, β_P), the

5. A call option/a put option, is the right to buy/to sell an asset at a specified exercise price on or before a specified expiration date (see, e.g., Bodie et al., 2005, Glossary).

household decisions (ω_P) and the monetary policy instrument (R_t).

Meanwhile, the equations (3.26) and (3.17) give rise to:

$$E(\varepsilon_{t+1}) = \frac{\beta_P}{\omega_P} \frac{\left[E\left(R^m_{t+1}\right) - R_t\right]}{R_t}, \tag{3.27}$$

where $E(\varepsilon_{t+1}) = (1 - E(\Sigma_{t+1}))$, denotes the expected nominal net spread rate between $E(R^{Sec}_t)$ and R_t. Thus, the expected net spread rate is increasing in the risk premium $\beta_P[E(R^m_{t+1}) - R_t]$, since an increase in the risk premium implies a strong contribution of risky assets to returns on the portfolio. However, the net spread rate is decreasing in the weight invested in risky assets, ω_P. In fact, an increase of ω_P means according to the model an increased diversification of the portfolio of risky assets. Therefore, the higher the portfolio of risky assets is diversified, the lower are its return $E(R^{Sec}_{t+1})$ and the net spread rate.

Moreover, one can derive from (3.26) the following equation:

$$\frac{\partial R^{Sec}_t}{\partial R^m_t} = \frac{\beta_P}{\omega_P}, \tag{3.28}$$

suggesting that the relation between stock market returns and securities returns depends on the sign of the beta coefficient of the portfolio. They move together for $\beta_P > 0$ but conversely when $\beta_P < 0$.

We obtain from (3.18) a negative relation between returns on securities and the risk-free rate such that:

$$\frac{\partial R_t}{\partial R^{Sec}_t} = -\frac{\omega_P}{1 - \omega_P} \qquad \text{where} \qquad \omega_P \neq 1, \tag{3.29}$$

implying that bonds become less attractive when returns on risky assets increase. Finally, based on the equations (3.28) and (3.29), the relation between the risk-free rate and stock market returns is given by

$$\frac{\partial R_t}{\partial R^m_t} = -\frac{\beta_P}{1 - \omega_P} \qquad \text{where} \qquad \omega_P \neq 1, \tag{3.30}$$

showing as in (3.29) that bonds become less attractive when stock market returns increase.

Therefore, the equations (3.28), (3.29) and (3.30) may help to capture how financial markets shocks impact on the real economy, through their influence on the monetary policy instrument and on households consumption and investment decisions in bonds and securities. On the other hand, they appear to be useful in analyzing the effects of a monetary policy shock, in an economy where we take in account the possibility for the household to acquire risky assets.

3.2.4 Central bank

We assume that the central bank systematically reacts to deviations of inflation, Π_t, and output growth, G_{Y_t}, from their steady-state values while smoothing short-term movements in the policy rate, R_t (see also Erceg and Levin, 2003; Galí and Rabanal, 2004; Liu and Phaneuf, 2007; El Omari and Phaneuf, 2011). Thus, monetary policy evolves according to the following Taylor-type policy rule:

$$log(R_t/R) = \rho_r log(R_{t-1}/R) + (1-\rho_r)\left[\rho_\pi log(\Pi_t/\Pi) + \rho_Y log(G_{Y_t}/G_Y)\right] + \epsilon_{R_t}, \tag{3.31}$$

where $G_{Y_t} = Y_t/Y_{t-1}$; R, Π, and G_Y are the steady-state values of R_t, Π_t, and G_{Y_t}, respectively; and ϵ_{R_t} is a monetary policy shock normally distributed with zero mean and standard deviation σ_R.

3.2.5 Markets clearing conditions

Market clearing on capital and labor markets requires respectively

$$K_t = \int_0^1 K_t^d(j)dj, \tag{3.32}$$

$$L_t = \int_0^1 L_t^d(j)dj. \tag{3.33}$$

The resource constraint of the economy implies that

$$Y_t = C_t + I_t. \tag{3.34}$$

Finally bonds and securities held by households are equal to zero, so $B_t = 0$ and $S_t^{ec} = 1$ in equilibrium.

3.3 Calibration

We calibrate the model parameters to match salient features of the U.S. economy.[6] Table 3.1 reports calibration values. We set the subjective discount factor β to 0.9926, implying an annual real interest rate at the steady-state of 3%. The parameter η, denoting the weight on leisure in the utility function is set equal to 1.315 (Christensen and Dib, 2008), so that the household spends around one third of its time in market activities. The inverse of the Frisch wage elasticity of labor supply χ is assigned the value of 1 (Dib, 2010; Falagiarda, 2013), implying an elasticity of intertemporal substitution of labor of 1. The depreciation rate of capital δ is calibrated to 0.025 (Christiano, Eichenbaum and Evans, 2005; Falagiarda, 2013), which implies an annual rate of depreciation on capital equal to 10%. We set the parameter of habit formation h to 0.8 (Fuhrer, 2000; Boldrin, Christiano and Fisher, 2001; Phaneuf and Tchakondo, 2013). The AR(1) coefficient of the productivity process ρ_a is set at 0.95, and its standard deviation σ_a is set at 0.45 (Smets and Wouters, 2007). The capital share in aggregate output production α is set at 0.33 (see, Ascari, 2004; Dib, 2010). The steady-state gross inflation rate γ is set equal to 1.0079 (see, Christensen and Dib, 2008).

The elasticity of substitution between differentiated goods θ determines the steady-state markup of prices over marginal cost, with a markup of $\theta/(\theta-1)$. Rotemberg and Woodford (1997) assume a value-added markup of 1.2, implying $\theta = 6$. Christiano, Eichenbaum and Evans (2005) estimate value-added markup at 1.2 in a model controlling for variable capital utilization. Nakamura and Steinsson (2010) assume $\theta = 4$ and a value-added markup 1.33 in a menu-cost model. So, we set $\theta = 6$, inducing a value-added markup of 1.2.

The parameter ξ_p, which measures the probability of price non-reoptimization, is fixed as follows. In a survey of postwar evidence on U.S. price behavior, Taylor (1999) documents that prices have changed about once a year on average. Using

6. See Appendix for detailed description of the dataset.

summary statistics from the Consumer Price Index micro data compiled by the U.S. Bureau of Labor Statistics for 350 categories of consumer goods and services, Bils and Klenow (2004) document that the median waiting time between price adjustments has been 4.3 months when price adjustments occuring during temporary sales are taken into account, while it has been 5.5 months when they are not. Their evidence, however, covers only a very short period of time, the years 1995-1997. Using a fewer categories of consumer goods and services, they report evidence suggesting that for the longer period 1959-2000 the frequency of price adjustments is much lower than for the years 1995-1997. Nakamura and Steinsson (2008) provide estimates of the frequency of price changes ranging from 8 to 11 months when product substitutions and temporary sales are both excluded, and from 7 and 9 months when only temporary sales are excluded.

In light of these studies, we set the value of ξ_p at 2/3 (see also Phaneuf and Tchakondo, 2012, 2013; El Omari and Phaneuf, 2011). Bils and Klenow (2004) emphasize the median as their measure of waiting time between price adjustments. Approximating the waiting time to the next price change by ξ_p^t, the median waiting time between price changes is given by $-\ln(2)/\ln(\xi_p)$.[7] Setting $\xi_p = 2/3$ implies that the median waiting time between price changes is 5.1 months, which is in the range of admissible values from micro level evidence.

The coefficients of the Taylor rule are calibrated as follows: $\rho_r = 0.8$, $\rho_\pi = 1.5$ and $\rho_Y = 0.125$. These values are broadly consistent with recent estimates reported in Smets and Wouters (2007) and Justiniano and Primeceri (2008), and with the calibration in Christiano, Eichenbaum and Evans (2005) and El Omari and Phaneuf (2011). The standard deviation of the monetary policy shock σ_R is set at 0.0004 (Ireland, 2007).

The calibration of the parameters related to financial markets is very tricky as the existing literature is uninformative. So, we set the volatility of stock market σ to 0.017, which corresponds to the quarterly mean of the volatility of stock price

7. See Cogley and Sbordone (2008, footnote 19).

index for U.S.(1977Q1 - 2010Q1), i.e., the 360-day standard deviation of the return on the national stock market index (Bloomberg). The market return rate μ is set at 0.03, which is equivalent to the mean value of the market return rate using quarterly data on SP500 (1980Q1 - 2013Q1). Using quarterly data from Fama-French benchmark factors returns (1930Q1 - 2014Q2), we set the beta coefficient of the portfolio β_P at 3.3 which corresponds to the mean value. For the weight invested in risky assets ω_P, we use a dataset from financial accounts of the United States for the period 1950Q1-2014Q3 [8]. Based on households treasury securities and corporate securities, we obtain a mean value of 0.9 for the weight invested in risky assets. Adding households savings deposits gives a mean value of 0.6 for the period which we set as a reasonable value of ω_P. Further, a sensitivity analysis is made with respect to some other values of β_P (-6, 5.5) and ω_P (0.4). Finally, we approximate the stock market shock to a sentiment shock, which reflects household beliefs about fluctuations in stock market bubbles (see Miao, Wang and Xu; 2012). Accordingly, we set the autocorrelation coefficient of the stock market process ρ_{R^m} at 0.87 and its standard deviation σ_{R^m} at 0.21.

3.4 Results

This section presents the results of the simulated model. On the one side, we present the effects of standard supply and demand shocks on some key macroeconomic variables. On the other side, we report their dynamic responses to the financial markets shock. Figures 3.1 and 3.2 show the impulse responses functions to technology and monetary policy shocks, respectively. Figures 3.3 - 3.6 plot the responses to the financial markets shock under various aspects.

3.4.1 Responses to technology shock

Figure 3.1 shows the impulse responses to a positive technology shock. Following this shock, in the basic model, output and consumption are increasing while

8. See Appendix A, for more details.

inflation and nominal interest rate decline. For the CAPM, it must be stressed that technology shock is perceived by financial markets as an element that may increase the profits of firms. Thus, the expected return on securities increases in reaction to the shock. This leads to a stronger decline in the nominal interest rate in the CAPM model since bonds are no longer attractive in that context. So, the increase in consumption, output and investment is lower in the CAPM model, because households invest in risky assets as anticipating higher future profits of firms. Therefore, compared to the basic model, the decline in inflation and labour is stronger in the CAPM model.

3.4.2 Responses to monetary policy shock

Figure 3.2 depicts the impulse responses to a contractionary monetary policy shock. In reaction to this shock, the nominal interest rate increases and output, consumption, inflation, investment and labor fall on impact. Following a tightening in monetary policy, returns on securities fall, since investors anticipate the conditions of economic activity that could limit future profits of firms. So, the decrease in such variables is higher in the CAPM model than in the basic model.

3.4.3 Responses to financial market shock

In what follows, we show the impulse responses to a positive financial markets shock. This financial markets shock can be interpreted as an exogenous increase in the return on the stock market. It may result from an information in financial markets or in the economy as a whole, which is anticipated or perceived by investors as something that could hit the stock market. The analysis is undergone through a series of cases. Basically, the purpose of this analysis is to understand how a financial markets shock could affect macroeconomic variables, especially its effects on the monetary policy instrument in our model economy.

Case 1. *For any $\beta_P > 0$, ω_P being constant, $\frac{\partial R_t^{S^{ec}}}{\partial R_t^m} > 0$ and $\frac{\partial R_t}{\partial R_t^m} < 0$. There exists a positive relation between R_t^m and $R_t^{S^{ec}}$, and a negative relation between*

R_t^m and R_t.

So, a positive financial markets shock, by increasing the return on the stock market induces the return on securities to increase as showed in Figure 3.3. Households could invest more to take advantage from higher future returns on risky assets. However, the return on bonds is expected to decrease as they become relatively less attractive for investors. Thus, the rise of future anticipated gains on risky assets combined with low bond yields should result in an increase in consumption, output, inflation, investment and labour in reaction to the financial markets shock.

Case 2. *Let $\beta_P > 0$, ω_P being constant. For $\beta_{P_1} > \beta_{P_2}$, $\frac{\partial R_t^{sec}}{\partial R_t^m}(\beta_{P_1}) > \frac{\partial R_t^{sec}}{\partial R_t^m}(\beta_{P_2})$ and $\left|\frac{\partial R_t}{\partial R_t^m}(\beta_{P_1})\right| > \left|\frac{\partial R_t}{\partial R_t^m}(\beta_{P_2})\right|$. The positive relation between R_t^m and R_t^{sec}, and the negative relation between R_t^m and R_t are increasing in β_P.*

This proposition can be seen as a corollary of **Case 1.**, since we have the same responses of our variables of interest following a financial markets shock. However, the analysis is undergone taking different values of β_P. Figure 3.4 shows that, the positive reaction of the return on securities and the negative response of the nominal interest rate to the financial markets shock are increasing in β_P for a fix weight invested in risky assets ω_P. In addition, the increase in consumption, output, inflation, investment and labour is more important for higher values of β_P. In fact, as β_P positively affects the expected return on the household portfolio of assets, higher profits are associated with higher value of β_P. This leads to a relative large increase in such macroeconomic variables in the case where β_P is higher.

Case 3. *Let $\beta_P > 0$ being constant. For $\omega_{P_1} > \omega_{P_2}$, $\frac{\partial R_t^{sec}}{\partial R_t^m}(\omega_{P_1}) < \frac{\partial R_t^{sec}}{\partial R_t^m}(\omega_{P_2})$ and $\left|\frac{\partial R_t}{\partial R_t^m}(\omega_{P_1})\right| > \left|\frac{\partial R_t}{\partial R_t^m}(\omega_{P_2})\right|$. The positive relation between R_t^m and R_t^{Sec} is decreasing in ω_P, while the negative relation between R_t^m and R_t is increasing.*

Here again, we deal with a corollary of **Case 1.**, but for different values of ω_P, β_P being constant. The positive reaction of the return on securities to the financial markets shock is decreasing in ω_P, while the negative response of the nominal interest rate is increasing for a given value of the beta coefficient of the portfolio β_P. Recall that, a higher value of ω_P means a more diversified portfolio that is associated with low risk and return according to the CAPM framework. So, the higher the value of ω_P, the higher the portfolio of risky assets is diversified and the lower is the return on securities. In addition, a higher value of ω_P supposes that the investor is more focused on risky assets than bonds. Hence, the lack of interest in bonds seems to be increasing in ω_P. Therefore, the decrease in return on risk-free assets following the shock is greater for higher values of ω_P. Finally, as showed in Figure 3.5, consumption, output, inflation, investment and labour are expected to be relatively less larger for a higher ω_P, since the expected profits here following the financial markets shock are lower.

Case 4. *For any $\beta_P < 0$, ω_P being constant, $\frac{\partial R_t^{Sec}}{\partial R_t^m} < 0$ and $\frac{\partial R_t}{\partial R_t^m} > 0$. There exists a negative relation between R_t^m and R_t^{Sec}, and a positive relation between R_t^m and R_t.*

A negative beta coefficient means that the portfolio is inversely correlated with the stock market. It turns out that, the return on securities decreases in reaction to a positive financial markets shock as we can see in Figure 3.6. As investors anticipate a market decline, the positive financial markets shock makes risky assets less attractive and thereby increases the attraction of investors for bonds. Thus, the nominal interest rate appears to be increasing. Moreover, the decline in the return on risky assets causes the return on the household portfolio to decrease, leading to the fall in consumption, output, inflation, investment and labour.

Overall, the CAPM that is subject to household portfolio choice decisions between risky and risk-free assets as proposed in this framework, modifies the effects of standard technology and monetary policy shocks on the dynamics of key macroeconomic variables. For instance, the CAPM model mitigates the responses of consumption, output, and investment, but amplifies the decline in the nominal interest rate, inflation and labor following a positive technology shock. On the other hand, the negative effects of a tightening monetary policy shock are amplified since the responses of output, consumption, inflation, investment and labor are stronger in the CAPM model than in the basic model.

In addition, a positive financial markets shock has different effects on the monetary policy instrument depending on values taken by the beta coefficient of the household portfolio β_P, and the weigh invested in risky assets ω_P. In particular, **Cases 1.,2.,3.** show a negative reaction in the nominal interest rate to the financial markets shock when β_P is positive for any value of omega. Here, consumption, output, inflation, investment and labor react positively. However, the nominal interest rate responds positively to the shock from **Case 4.**, where β_P is negative and consumption, output, inflation, investment and labor decrease. From the monetary policy perspective, **Cases 1.,2.,3.** suggest that an upturn in financial markets is expected to exert a downward pressure on the monetary policy instrument in our model economy. It turns out that, a rise in financial markets not accompanied by a reaction of the central bank may result in lower nominal interest rates. The monetary authority should then increase the nominal interest rate following the positive financial markets shock in order to bring down inflation. Conversely, a decline in financial markets gives rise to an increase in the nominal interest rate. The central bank can then lower its policy rate to boost the economy. In short, the central bank should increase the nominal interest rate when financial markets are in a strong momentum of rising and lower it when they are experiencing sharp declines.

As a consequence, these results suggest that financial markets, particularly stock markets might deserve special attention from policy-makers and researchers when they assess the ins and outs of monetary policy.

3.4.4 Volatilities and autocorrelations

Here, we assess the capacity of the CAPM model that accounts for households' assets portfolio choice decisions to replicate some important features of the U.S. macroeconomic fluctuations. As in Christensen and Dib (2008), and Dib (2010), we pay attention to the model-implied volatilities (standard deviations), relative volatilities, and correlations of output with some key variables. Table 3.2 reports the standard deviations and relative volatilities of output, consumption, investment, labor, and inflation from the data, and for the two simulated models. [9] The standard deviations are expressed in percentage terms. The model-implied standard deviations, relatives volatilities and unconditional correlations with output are calculated using technology and monetary shocks.

Column 3, in Table 3.2 displays standard deviations, relative volatilities, and unconditional autocorrelations of the actual data for the period 1954Q1-2008Q3. Columns 4 and 5 however, report those simulated with the basic and CAPM models, respectively. In the data, the standard deviation of output is 1.54, consumption 1.22, and investment 7.08. Labour has a standard deviation of 1.76. Inflation is less volatile with a standard deviation of 0.38. In addition, investment and labor are 4.60 and 1.14 times as volatile as output, while consumption and inflation are less volatile than output, with relative volatilities of 0.79 and 0.25 respectively. In addition, output, consumption, investment and labor are highly persistent, with autocorrelations coefficients that are, at least, equal to 0.8; inflation is less so, with a coefficient of 0.44.

The simulation results show that, in the CAPM model, all volatilities are close to those in the data. However, the basic model, in which the financial market sector is absent, overpredicts all the volatilities. Compared to the basic model, the CAPM model is also successful at matching the relative volatility of most of the variables. Moreover, the CAPM model does a better job at matching the autocorrelations shown in the data.

Finally, Table 3.3 displays the cross-correlations of the data and those simulated in the two models. Overall, the correlations of output with consumption, investment, labor, and inflation implied by the CAPM model fit better those observed in the data.

9. All series in the data are HP-filtered before calculating their standard deviations as well as their unconditional autocorrelations and cross-correlations

3.5 Conclusion

The aim of this paper is to improve our understanding of the interrelations between financial markets and the real economy. We propose a DSGE framework that embeds a financial markets sector based on geometric Brownian motion (GBM) to describe stock markets prices and returns. We also incorporate the capital asset pricing model (CAPM) to account for the behaviour of investors on stocks markets in terms of purchases of equities. The contributions of this paper are twofold. First, following a technology shock, consumption, output and investment respond less here, while inflation and labor are reacting more strongly than the baseline model where financial markets are ruled out. In addition, the negative effects of a tightening monetary policy shock on output, consumption, inflation, investment and labor are more significant. Second, a positive financial markets shock negatively affects the nominal interest rate when the beta coefficient of the household portfolio of assets is positive. In that situation, a lack of reaction of the central bank may result in a downward pressure on the nominal interest rate. Accordingly, our results suggest that stock markets might deserve special attention from policy makers and researchers when they analyze the ins and outs of monetary policy.

We acknowledge the model developed here suffers the limitations and shortcomings associated with the basic CAPM that is one of the main pillars of our framework. Thus, further works could extend the analysis by taking account of the three-factor model for expected returns proposed by Fama and French (1993, 1996, 2004). Doing so may quantitatively affect some of our results, but the intuition developed here will still remain in force.

Parameter	Value	
Subjective discount factor	$\beta = 0.9926$	
Weight on leisure in the utility function	$\eta = 1.315$	
Frisch elasticity of labor supply	$\chi = 1$	
Capital depreciation rate	$\delta = 0.025$	
Habit formation parameter	$h = 0.8$	
Technology process	$\rho_a = 0.95,$	$\sigma_a = 0.45$
Capital input share	$\alpha = 0.33$	
Trend inflation	$\gamma = 1.0075$	
Elasticity of substitution between differentiated goods	$\theta = 6$	
Probability of price non-reoptimization	$\xi_p = 2/3$	
Monetary policy parameters	$\rho_r = 0.8,$	$\rho_\pi = 1.5$
	$\rho_Y = 0.125,$	$\sigma_R = 0.0004$
Stock market volatility	$\sigma = 0.017$	
Stock market return rate	$\mu = 0.03$	
Portfolio beta coefficient	$\beta_P = 3.3$	
Risky assets weight	$\omega_P = 0.6$	
Stock market process	$\rho_{R^m} = 0.87,$	$\sigma_{R^m} = 0.21$

Table 3.1 Calibrated Parameters Values

Variables	Definitions	Data	Basic	CAPM
	A. Standard deviations (in %)			
Y_t	output	1.54	1.74	1.65
C_t	consumption	1.22	1.51	1.25
I_t	investment	7.08	8.25	7.69
L_t	labor	1.76	2.82	1.82
π_t	inflation	0.38	0.59	0.35
	B. Relative volatilities			
Y_t	output	1	1	1
C_t	consumption	0.79	0.87	0.76
I_t	investment	4.60	4.74	4.66
L_t	labor	1.14	1.62	1.10
π_t	inflation	0.25	0.34	0.21
	C. Autocorrelations			
Y_t	output	0.84	0.92	0.85
C_t	consumption	0.85	0.94	0.90
I_t	investment	0.80	0.85	0.82
L_t	labor	0.90	0.73	0.88
π_t	inflation	0.44	0.33	0.41

Table 3.2 Standard Deviations and Relative Volatilities (Data: 1954Q1-2008Q3)

Variables	Definitions	Data	Basic	CAPM
Y_t	output	1	1	1
C_t	consumption	0.87	0.16	0.84
I_t	investment	0.91	0.73	0.82
L_t	labor	0.87	0.88	0.86
π_t	inflation	0.15	0.89	0.20

Table 3.3 Correlations with output (Data: 1954Q1-2008Q3)

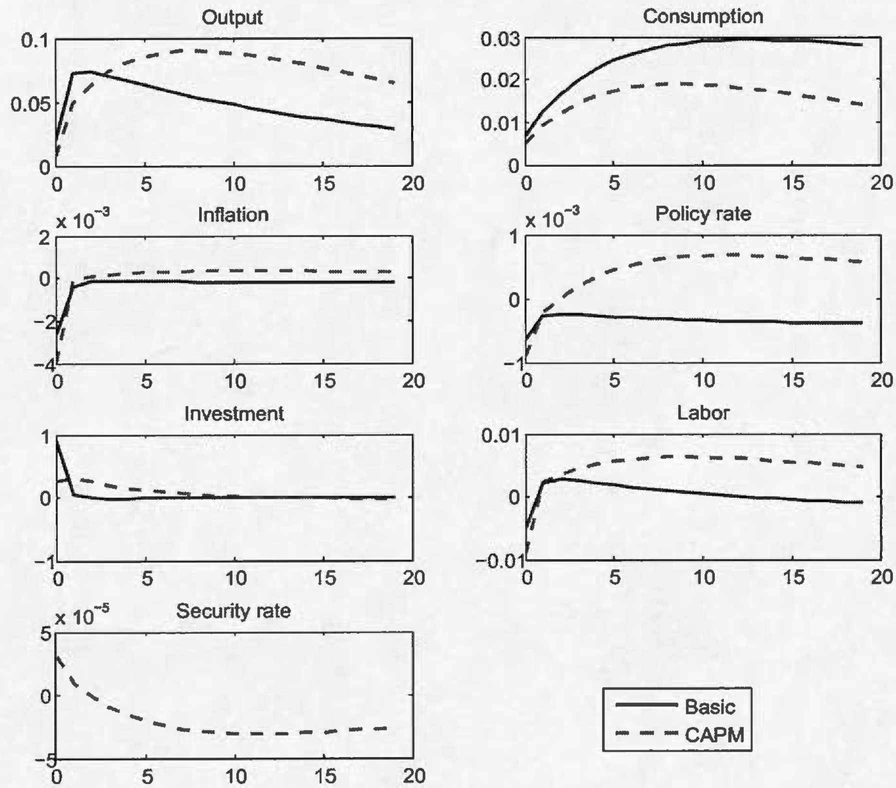

Figure 3.1 IRFs to a Positive Technology Shock for $\beta_p = 3.3$ and $\omega_p = 0.6$.

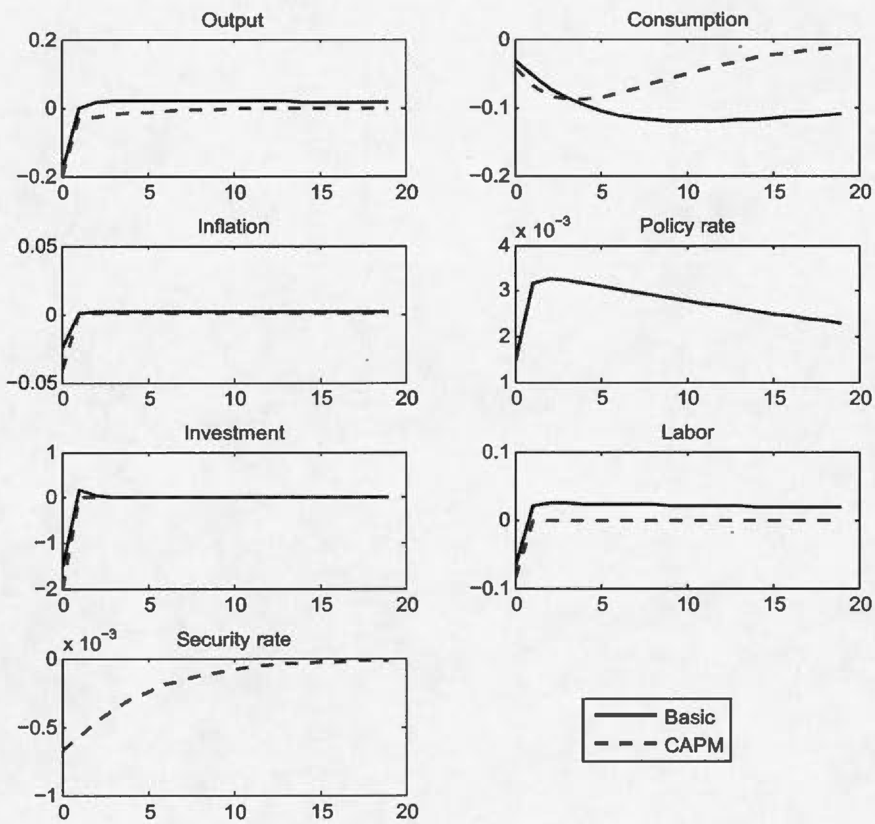

Figure 3.2 IRFs to a Contractionary Monetary Policy Shock for $\beta_p = 3.3$ and $\omega_p = 0.6$.

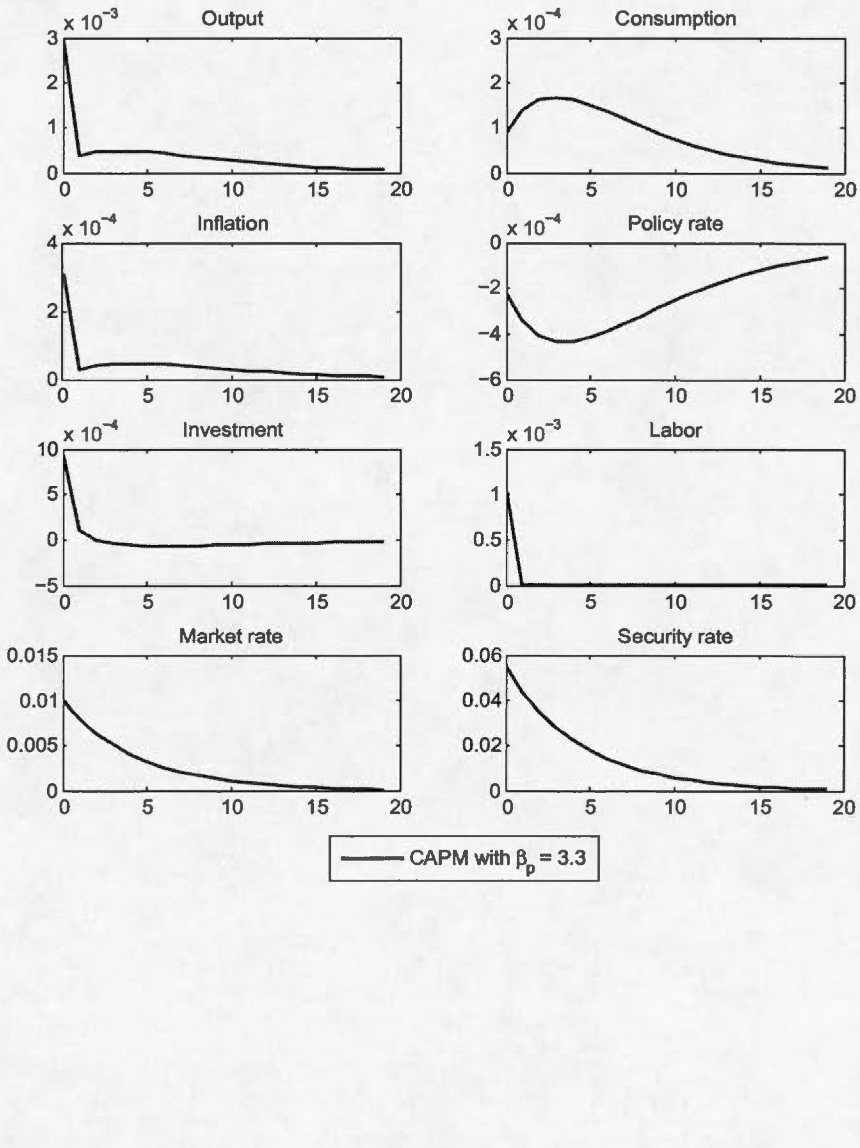

Figure 3.3 IRFs to a Positive Financial Market Shock for $\beta_p > 1$ and $\omega_p = 0.6$.

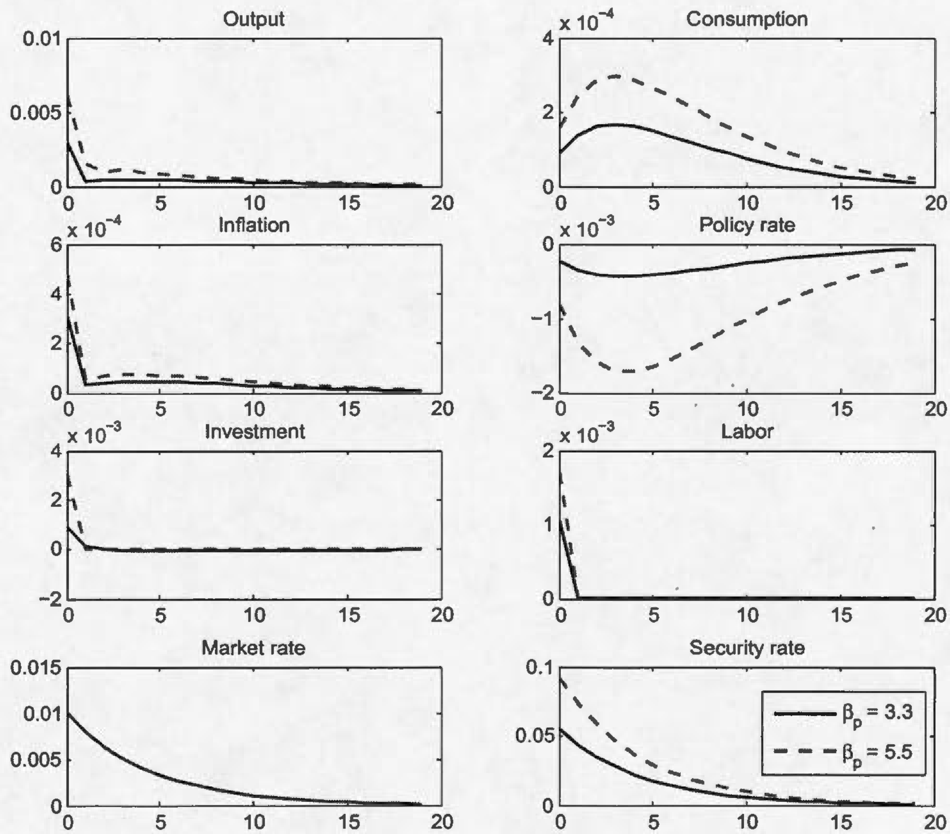

Figure 3.4 IRFs to a Positive Financial Market Shock for $\beta_p = 3.3$, $\beta_p = 5.5$ and $\omega_p = 0.6$.

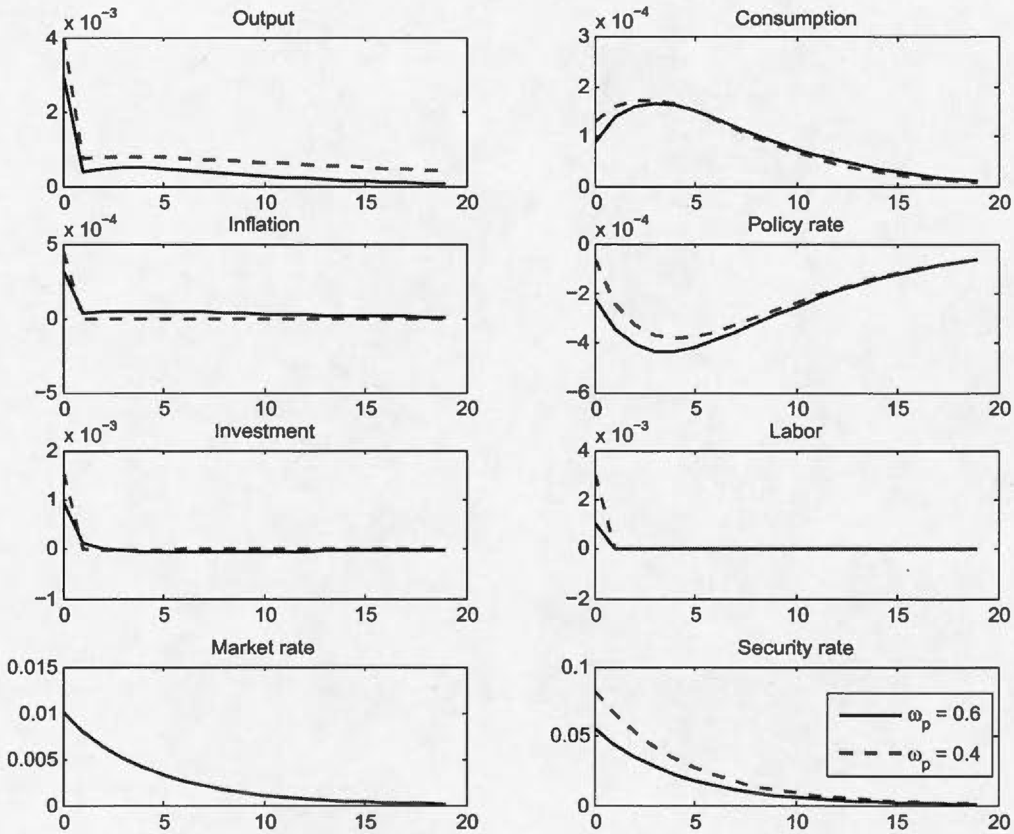

Figure 3.5 IRFs to a Positive Financial Market Shock for $\omega_p = 0.6$, $\omega_p = 0.4$ and $\beta_p = 3.3$.

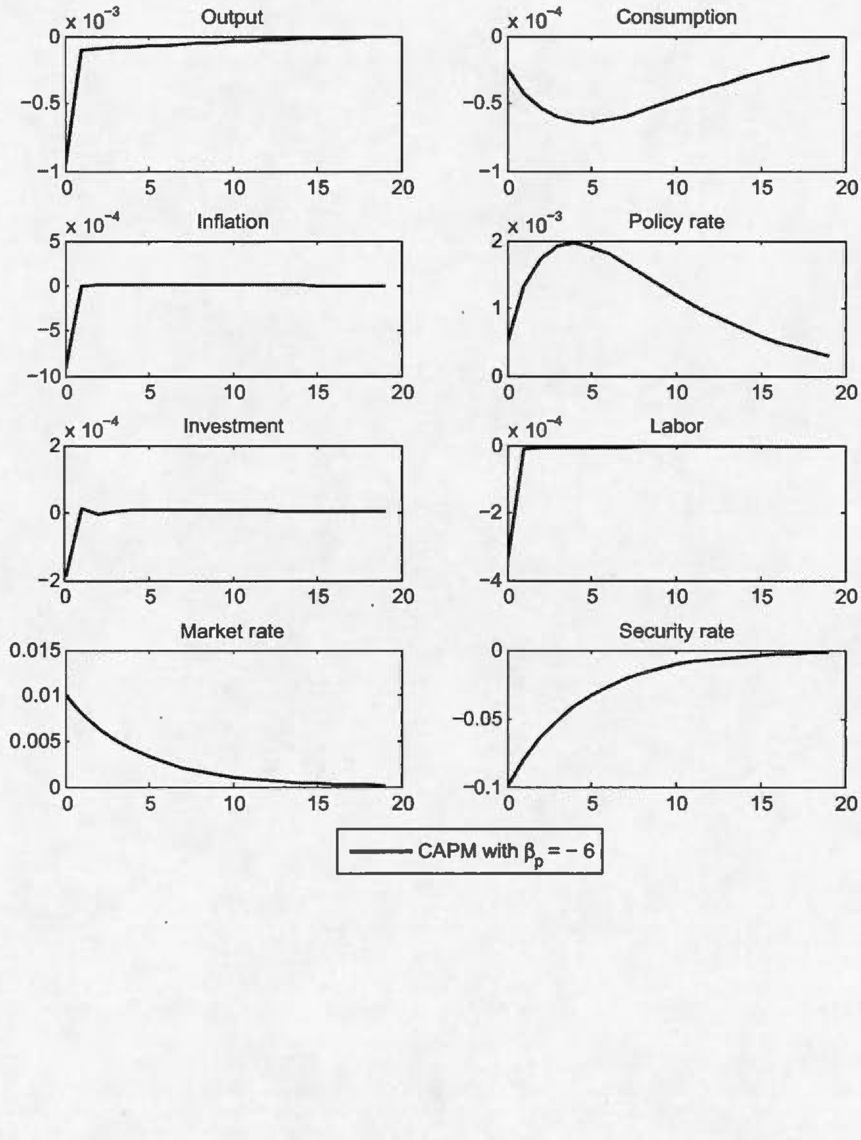

Figure 3.6 IRFs to a Positive Financial Market Shock for $\beta_p < 0$ and $\omega_p = 0.6$.

CONCLUSION

Dans cette thèse, nous analysons les dynamiques à court terme de l'inflation et les interrelations entre les marchés financiers et l'économie réelle. Le but étant ici de contribuer à une meilleure application de la politique monétaire. Les deux premiers chapitres montrent la pertinence de la structure en boucle de production dans l'explication de la persistance de l'inflation observée dans les données. Le troisième chapitre souligne l'importance de la prise en compte des marchés boursiers afin de mieux comprendre l'impact des chocs technologique, monétaire et financiers sur les variables macroéconomiques.

L'objectif du premier chapitre était de dériver une nouvelle courbe de Phillips néo-keynésienne (NKPC), à travers un modèle DSGE où nous incorporons les inputs intermédiaires et le trend d'inflation positif. Comme résultat principal, nous trouvons que les biens intermédiaires semblent avoir un effet plus important sur la pente de la NKPC que le trend d'inflation positif.

Cette analyse est approfondie dans le deuxième chapitre où nous simulons un modèle DSGE en présence toujours des deux ingrédients, afin de confronter les autocorrélations de l'inflation du modèle à celles observées dans les données américaines. Nous trouvons ici que les biens intermédiaires donnent une meilleure explication de l'évidence empirique sur la persistance de l'inflation que ne le fait le trend d'inflation positif, confortant ainsi les conclusions du premier chapitre.

Enfin, la contribution du dernier chapitre est une modélisation explicite d'un secteur de marchés financiers, plus particulièrement, le marché boursier dans les modèles DSGE. Pour ce faire, nous tenons compte des décisions des ménages sur les marchés financiers grâce au modèle d'évaluation des actifs financiers (CAPM) de Fama et French (2004). Puis, les dynamiques du marché boursier sont décrites en s'appuyant sur le mouvement brownien géométrique. Ainsi, nous montrons que nos connaissances standard des effets des chocs technologique et de politique monétaire sur les variables macroéconomiques semblent être modifiées dans ce nouveau cadre d'analyse. Nous suggérons aussi qu'un choc au marché financier a des conséquences non négligeables sur le taux d'intérêt nominal, qui est par ailleurs, l'instrument de politique monétaire. En outre, notre cadre d'analyse reproduit mieux certaines caractéristiques clés du l'économie U.S., en l'occurrence, les volatilités et autocorrélations des principales variables macroéconomiques ainsi que leurs corrélations avec l'output.

APPENDIX A

FINANCIAL MARKETS AND THE CAPITAL ASSET PRICING MODEL IN A DSGE FRAMEWORK

Data sourced from St. Louis Fed, and Kenneth French's data library on the official Tuck at Dartmouth MBA school home page.

1. Stock market prices are measured by Standard and Poor 500 Index (SP500).
2. The stock market volatility is measured by the Volatility of Stock Price Index for United States (DDSM01USA066NWDB).
3. Stock market return rates are measured using Fama/French Benchmark Factors, CRSP Quarterly data.
4. Financial Accounts of the United States; Federal Reserve Statistical Release. Z.1 Statistical Release for Dec 11, 2014; L.205 (Q), L.209 (Q), L.213 (Q).

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